Recursive

Long number mult → correctness (MI) ~~A~~ base → —O—
→ size 2 →
→ size 4 →
→ size 8 →

Long number mult → Run time (tree of rec. calls)

$X = A \cdot 2^{\frac{n}{2}} + B$

$Y = C \cdot 2^{\frac{n}{2}} + D$

$A * C$ coeff 2^n

$B * D$ coeff of 1

$(A+B) * (C+D) - A*C - B*D$ coeff $2^{n/2}$

3 multiplications

```
M2 ( X,Y) = T(n)                                   n
    n=size(X);                                     1
    if (n==1) return Lut[X][Y];                    n
    std::tie(A,B)=dissociate(X);                   n
    std::tie(C,D)=dissociate(Y);                   T(n/2)
    m1 = M2(A,C);                                  T(n/2)
    m4 = M2(B,D);
    m23 = M2(A+B, C+D) - m1 - m4                   T(n/2 +1) + 2n
    return m1 << n + m23 << n/2 + m4;
}
```

$$T(n) = \underline{\underline{3}}\, T\left(\frac{n}{2}\right) + 8n + 3 \quad \text{simplify}$$

$$\begin{cases} T(n) = 3\, T\left(\frac{n}{2}\right) + n \\ T(1) = 1 \end{cases}$$

ℓ

$h = \log_2 n$

$\cancel{T(n)}$

$\cancel{T(\frac{n}{2})}$ $\cancel{T(\frac{n}{2})}$ $\cancel{T(\frac{n}{2})}$

$T(\frac{n}{4})$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $T(\frac{n}{4})$

$T(\frac{n}{8})$

$T(\frac{n}{16})$

$T(1)$. - - - - - - - . . $T(1)$

0	n
1	$3\frac{n}{2}$ $\boxed{\cancel{+\frac{n}{2}+\frac{n}{2}}}$
2	$9\frac{n}{4}$
3	$27\frac{n}{8}$
k	$3^k\frac{n}{2^k}$
$h-1$	
h	

$b=3$

$h=\log_2 n \Leftarrow \frac{n}{2^x}=1$

$$R = b^h = 3^{\log_2 n} = n^{\log_2 3} \quad (\approx n^{1.57})$$

$$C = \sum_{k=0}^{h-1} \left(\frac{3}{2}\right)^k n = n\left(1 + \frac{3}{2} + \left(\frac{3}{2}\right)^2 + \dots + \left(\frac{3}{2}\right)^{h-1}\right) = \left[\begin{array}{c}\text{apple} \\ \text{geom.} \\ \text{series} \\ \text{sum} \\ \text{form}\end{array}\right]$$

$$= n \frac{\left(\frac{3}{2}\right)^h - 1}{\cancel{\frac{3}{2} - 1} \; \frac{1}{2}} = 2n\left(\frac{3^h}{2^h} - 1\right) =$$

$$= 2n\left(\frac{n^{\log_2 3}}{n} - 1\right) = 2n^{\log_2 3} - 2n$$

$$T(n) = R + C = \cancel{3n\log}$$

$$= 3n^{\log_2 3} - 2n = O\left(n^{\log_2 3}\right)$$

faster $O(n^2)$

M2(X, Y)

if(n==1) retur eu

time

m1 = M2 (...)

down

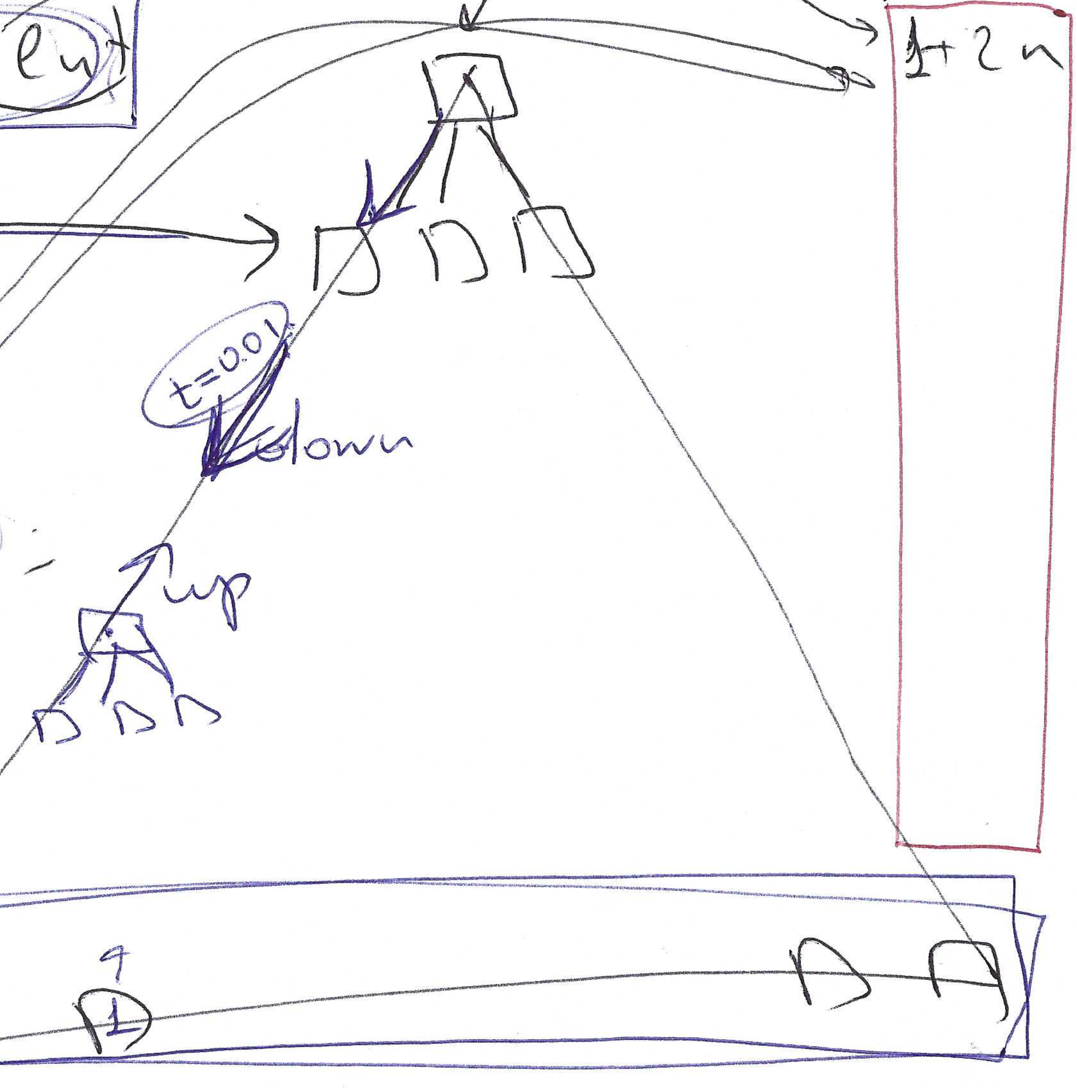
m2 = M2 (...)

m3 = M2 (...)

return

up

m1 + m2 + m3 ;



Brute-force

1) ~~stupid~~ no!!
2) small problem $n = 5$
3) 1-time problem $\overbrace{\underset{\text{impl.}}{5\text{ min}} + \underset{\text{RT}}{12\text{ ho.}}}^{\text{BF}}$ $\overbrace{\underset{\text{impl.}}{20\text{h}} + \underset{\text{RT}}{1\text{M}}}^{\text{advanced}}$
4) first attempt to solve:
   a) get familiar
   b) learn the problem
   c) this will be your "~~test~~" checker?

unit test

STL

~~mutating~~ $\Rightarrow$ reorder $\supset$ sort

sort $\rightarrow$ unshuffles.

find a permutation that makes it sorted

permutation

$$\begin{array}{c c} \text{old} & \multirow{2}{*}{$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 2 & 3 & 0 & 4 & 1 \end{pmatrix}$} \\ \text{new} & \end{array} \quad \text{or} \quad (2\ 3\ 0\ 4\ 1)$$

$$\overset{0}{a}\ \overset{1}{b}\ \overset{2}{d}\ \overset{3}{e}\ \overset{4}{f} \xrightarrow{\text{perm}} \overset{0}{d}\ \overset{1}{f}\ \overset{2}{a}\ \overset{3}{b}\ \overset{4}{e}$$

number of permutations

$$0\ 1\ 2\ 3\ \ldots\ N-1$$

$$N * (N-1) * (N-2) * \ldots * 1 = N!$$

Opt. BF

candidate solution
feasible solutions

```
bsf = ±∞?/.....
for each A in [possible answers] {
    if (A is better) bsf = A;
}
return bsf;
```

0 ... 9
(0,7), (0,7)
~~(2,7)~~ (2,5)
~~(3,11)~~
~~(1,2,7)~~

Find BF

```
for each A in possible answers {
    if ( A satisfies ..... ) return A;
}
return NULL; // fail
```

early return

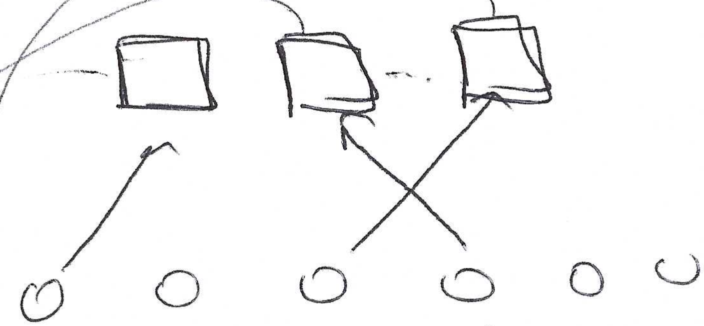
Combinatorics

permutations / shuffle

0 1 2 3

2 0 1 3

assignments



k pos

n values

assign n to n = perm

$$n(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!} = A(n,k)$$

$n = 0 \dots 9$

$k = 3$

1 2 3

3 2 1

Combinations = assignment, no order.

= n objects

SWAG with k objects

(1,2,3) ~~assign~~ (2,3,1) … (3,2,1).

Binomial

$$\binom{n}{k} = C(n,k) = \frac{A(n,k)}{k!} = \frac{n!}{(n-k)!\,k!}$$