Final Exam

Math 258
(Fall 2005)

Solve the following problems. Show all your work in the space under each problem.

1. How many subsets of an 8-element set have more than three elements? (2 pts)
   (*Hint*: It would be easier to calculate first those with three or fewer elements)

2. What is the coefficient of $x^4 y^3$ in $(2x + y)^7$, without doing the expansion? (2 pts)

3. Given a family with two children, what is the probability that both children are girls if the older child is a girl? (2 pts)

4. Determine whether the relation $R = \{(x, y) \in Z \times Z \mid x = y^2\}$ is: (3 pts)

   (a) reflexive (b) symmetric (c) antisymmetric

5. Given the relations: $R_1 = \{(x, y) \in R \times R \mid x > y\}$, $R_2 = \{(x, y) \in R \times R \mid x \geq y\}$ and $R_3 = \{(x, y) \in R \times R \mid x = y\}$, find the following: (4 pts)

   (a) $R_1 \cup R_2$ (b) $R_1 \cap R_3$

   (c) $R_2 - R_3$ (d) $R_1 \circ R_3$

6. A relation R on a set A is called **irreflexive** if **(** $x \in A \Rightarrow (x, x) \notin A, \forall x \in A$ **)**. In other words, no element in A relates to itself. Suppose now that a relation R is irreflexive. Is R^2 necessarily irreflexive? Explain. (2 pts)

7. Determine whether the following relation on the set of all functions from Z to Z is an equivalence relation. Check all the relevant properties: (3 pts)

$$R = \{(f, g) \mid f(0) = g(0) \text{ or } f(1) = g(1)\}$$

8. Find the equivalence relation induced by the following partition of the set S = {0,1,2,3,4,5}: (2 pts)

$$\{0\}, \{1,2\}, \{3,4,5\} .$$

   Are elements 2 and 3 equivalent in this relation?