

# HOMEWORK 1

## (Math 200-Section A)

1. Find a vector of magnitude 3 in the direction opposite to the direction of the vector  $\mathbf{u} = 2i - k$ . (2pts)
2. Find the center and radius of the sphere  $3x^2 + 3y^2 + 3z^2 + 2y - 2z = 9$ . (2 pts)
3. Find the length and direction of  $\mathbf{v} \times \mathbf{u}$ , where  $\mathbf{u} = i \times j$  and  $\mathbf{v} = j \times k$ . (2 pts)
4. Find the volume of the parallelepiped determined by the vectors  $\mathbf{u} = i + j - 2k$ ,  $\mathbf{v} = -i - k$  and  $\mathbf{w} = 2i + 4k - 2k$ . (2 pts)
5. Find the points in which the line  $x = 2 - t, y = t + 1, z = t - 3$  meets the coordinate planes. (2 pts)
6. Show that if  $\mathbf{r}(t) = f(t)i + g(t)j + h(t)k$  is differentiable at  $t_0$ , then it is continuous at  $t_0$  as well. Is the converse true? If not, give a counterexample. (2 pts)
7. Show that if  $\mathbf{u}, \mathbf{v}$  and  $\mathbf{w}$  are differentiable vector functions of  $t$ , then: (2 pts)

$$\frac{d}{dt} \det \begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix} = \det \begin{pmatrix} \frac{du_1}{dt} & \frac{du_2}{dt} & \frac{du_3}{dt} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix} + \det \begin{pmatrix} u_1 & u_2 & u_3 \\ \frac{dv_1}{dt} & \frac{dv_2}{dt} & \frac{dv_3}{dt} \\ w_1 & w_2 & w_3 \end{pmatrix} + \det \begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ \frac{dw_1}{dt} & \frac{dw_2}{dt} & \frac{dw_3}{dt} \end{pmatrix}$$

8. Find the length of the curve  $\mathbf{r}(t) = (3t)i + (3t)j + (2-t)k$  from  $(0, 0, 2)$  to  $(3, 3, 1)$ . (2 pts)
9. Find the equation of the circle of curvature of the curve  $\mathbf{r}(t) = ti + (\sin t)j$  at the point  $(\frac{\pi}{2}, 1)$ . (2 pts)
10. Find  $\mathbf{T}, \mathbf{N}, \mathbf{B}, \kappa$  and  $\tau$  for the curve  $\mathbf{r}(t) = (\frac{t^3}{3})i + (\frac{t^2}{2})j, t > 0$ . (2 pts)