HOMEWORK 1

(Math 200-Section B)

1. Find a vector of magnitude 2 in the direction opposite to the direction of the vector $\mathbf{u} = i - 2k$. (2pts)

2. Find the center and radius of the sphere $2x^2 + 2y^2 + 2z^2 + x - y + 2z = 5$. (2 pts)

3. Find the length and direction of $\mathbf{v} \times \mathbf{u}$, where $\mathbf{u} = i \times j$ and $\mathbf{v} = j \times k$ **.** (2 pts)

4. Find the volume of the parallelepiped determined by the vectors $\mathbf{u} = 2i + j$, $\mathbf{v} = 2i - j + k$ and $\mathbf{w} = i + 2k$. (2 pts)

5. Find the points in which the line $x = 3 - t, y = 2t - 1, z = 2t - 2$ meets the coordinate planes. (2 pts)

6. Show that if $\mathbf{r}(t) = f(t)i + g(t)j + h(t)k$ is differentiable at t_0 , then it is continuous at t_0 as well. Is the converse true? If not, give a counterexample. (2 pts)

7. Show that if **u**, **v** and **w** are differentiable vector functions of t, then: (2 pts)

$$\frac{d}{dt}\det\begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix} = \det\begin{pmatrix} \frac{du_1}{dt} & \frac{du_2}{dt} & \frac{du_3}{dt} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix} + \det\begin{pmatrix} u_1 & u_2 & u_3 \\ \frac{dv_1}{dt} & \frac{dv_2}{dt} & \frac{dv_3}{dt} \\ w_1 & w_2 & w_3 \end{pmatrix} + \det\begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ \frac{dw_1}{dt} & \frac{dw_2}{dt} & \frac{dw_3}{dt} \end{pmatrix}$$

8. Find the length of the curve $\mathbf{r}(t) = (2t)i + (2t)j + (1-t)k$ from (0, 0, 1) to (2, 2, 0). (2 pts)

9. Find the equation of the circle of curvature of the curve $\mathbf{r}(t) = ti + (\sin t)j$ at the point $(\frac{\pi}{2}, 1)$. (2 pts)

10. Find **T**, **N**, **B**, κ and τ for the curve $\mathbf{r}(t) = (\frac{t^3}{3})i + (\frac{t^2}{2})j$, t>0. (2 pts)