HOMEWORK 1

(Math 258)

1. Construct the truth table for each of the following compound propositions and determine whether they are equivalent. Is any one of them a tautology? (4 pts)

   (a) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$ (b) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

2. Let $P(x)$ be the statement "$x = x^2$." If the universe of discourse consists of the integers, what is the truth value of the following? Explain. (2 pts)

   (a) $P(0)$ (b) $P(-1)$ (c) $\exists x P(x)$ (d) $\forall x P(x)$

3. Determine the truth value of the following statements, if the universe of discourse consists of all real numbers. Explain. (4 pts)

   (a) $\forall x \exists y (x^2 = y)$ (b) $\forall x \exists y (x = y^2)$ (c) $\forall x \exists y (xy = 0)$ (d) $\exists x \exists y (x + y \neq y + x)$

4. Prove or disprove that product of a non-zero rational number and an irrational number is irrational. (*Hint*: Use the fact that the product of two rational numbers is rational) (2 pts)

5. Determine which of the following sets is the power set of a set. For the ones that are power sets, write down the set. (2 pts)

   (a) $\varnothing$ (b) $\{\varnothing, \{a\}\}$ (c) $\{\varnothing, \{a\}, \{\varnothing, a\}\}$ (d) $\{\varnothing, \{a\}, \{b\}\{a, b\}\}$

6. The **symmetric difference** of two sets A and B, denoted by $A \oplus B$, is the set containing those elements that belong to either A or B, but not in both. (6 pts)

   (a) Show that if $A \oplus B = A$, then $B = \varnothing$.

   (b) Prove or disprove: If $A \oplus C = B \oplus C$, then $A = B$.