

HOMWORK 2

(Math 200-Section A)

- Find the $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 - xy^2}{x^2 + y^2}$ by using the substitution: $x = r \cos \theta$, $y = r \sin \theta$. Notice that $r = \sqrt{x^2 + y^2}$, hence the limit will be taken over r now, instead of x and y .
(2 pts)
- The 3-dimensional **Laplace equation** is given by $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$. Show that the function $f(x) = 2z^3 - 3(x^2 + y^2)z$ is a solution to the Laplace equation. (3 pts)
- Use the Chain Rule to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$, at $(u,v) = (-1,2)$, of $w = xy + \ln z$, where $x = \frac{v^2}{u}$, $y = u + v$ and $z = \cos u$.
(3 pts)
- Is there a direction u in which the rate of change of $f(x) = x^2 - 3xy + 4y^2$ at $P = (1,2)$ equals 14? Give reasons for your answer.
(3 pts)
- The derivative of $f(x, y, z)$ at a point P is greatest in the direction of $v = i - j + k$. In this direction, the value of the derivative is $\sqrt{3}$. What is ∇f at P ?
(3 pts)
- Assuming that the necessary derivatives of $f(x, y, z)$ are defined, how are $D_i f$, $D_j f$ and $D_k f$ related to f_x , f_y and f_z ?
(3 pts)
- Find all the (local) maximum, minimum and saddle points of the function:
$$f(x) = x^3 + 3xy + y^3.$$

(3 pts)