HOMEWORK 2

(Math 200-Section B)

1. Find the $\lim\limits_{(x,y)\to(0,0)} \cos(\frac{x^3 - y^3}{x^2 + y^2})$ by using the substitution: $x = r\cos\vartheta,\ y = r\sin\vartheta$.
   Notice that $r = \sqrt{x^2 + y^2}$, hence the limit will be taken over *r* now, instead of x and y.
   (2 pts)

2. The 3-dimensional **Laplace equation** is given by $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$. Show that the function $f(x) = 2z^3 - 3(x^2 + y^2)z$ is a solution to the Laplace equation. (3 pts)

3. Use the Chain Rule to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$, at (*u*,*v*) = (-1,2), of $w = xy + \ln z$, where $x = \frac{v^2}{u}$, $y = u + v$ and $z = \cos u$. (3 pts)

4. Is there a direction *u* in which the rate of change of $f(x) = x^2 - 2xy + 3y^2$ at P = (2,1) equals 3? Give reasons for your answer. (3 pts)

5. The derivative of $f(x, y, z)$ at a point P is greatest in the direction of $v = i + j - k$. In this direction, the value of the derivative is $2\sqrt{3}$. What is ∇f at P ? (3 pts)

6. Assuming that the necessary derivatives of $f(x, y, z)$ are defined, how are $D_i f$, $D_j f$ and $D_k f$ related to f_x, f_y and f_z? (3 pts)

7. Find all the (local) maximum, minimum and saddle points of the function:

$$f(x) = x^3 + 3xy + y^3.$$

(3 pts)