HOMEWORK 2

(Math 258)

1. Find the prime factorization of 126. (2 pts)

2. The **Euler function φ** is the function defined as follows: (4 pts)

$$\varphi(n) = \#\{\, b \in Z \mid 0 < b \le n,\ \gcd(b, n) = 1\} ,\ n \in Z .$$

(a) Find $\varphi(10)$ (b) Show that n is a prime if and only if $\varphi(n) = n - 1$.

3. Find the gcd and lcm of the following pair of integers: (2 pts)

$$2^2 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11^2 \cdot 13 \ \text{ and } \ 2^3 \cdot 3^5 \cdot 11 \cdot 17^{13}$$

4. Show that if a,b,k and m are integers such that $k \ge 1, m \ge 2$ and $a \equiv b \bmod m$, then $a^k \equiv b^k \bmod m$. (2 pts)
(*Hint*: Use the identity for $a^k - b^k$).

5. Use the *extended Euclidean algorithm* to write the gcd(35, 78) as a linear combination of 35 and 78. Use that relation to find the inverse of 35 in Z_{78}. (4 pts)

6. Solve the congruence $4x \equiv 5 \bmod 9$. List at least three integers that are solutions of the congruence. (2 pts)

7. Solve the system of congruences: (4 pts)

$$x \equiv 2 \bmod 3$$
$$x \equiv 1 \bmod 4$$
$$x \equiv 3 \bmod 5.$$