HOMEWORK 3

(Math 200-Section B)

1. Find the radius and interval of convergence for the power series: (9 pts)

   (a) $\sum_{n=1}^{\infty} \frac{1}{n2^n} x^n$ (b) $\sum_{n=1}^{\infty} n^n x^n$ (c) $\sum_{n=1}^{\infty} \frac{3^n}{n!} x^n$

   (*Hint*: For (b), keep in mind that $\lim_{n\to\infty}(1+\frac{1}{n})^n = e$)

2. The power series for $\ln(1+x)$ is given by: (3 pts)

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad , \ -1 < x \le 1.$$

   Use this to approximate $\ln 2$ in two decimal places.

   (*Note*: Just use the first four terms of the power series)

3. **Term-by-term differentiation and integration:** (8 pts)

   If $f(x) = \sum_{n=0}^{\infty} a_n x^n$ **,** $-R < x < R$**, then:**

   (i) $f(x)$ **is differentiable and** $f'(x) = \sum_{n=0}^{\infty} \frac{d}{dx}(a_n x^n)$ **,** $-R < x < R$

   (ii) $f(x)$ **is integrable and** $\int f(x)dx = \sum_{n=0}^{\infty} a_n \frac{x^{n+1}}{n+1}$ $+ C$**,** $-R < x < R$**.**

   (a) Given that the power series of $\sin x$ is $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ and using the information above, find the power series for $\cos x$ (write down only the first four terms). Use that to show that $\frac{\sqrt{3}}{2} = 1 - \frac{\pi^2}{72} + \frac{\pi^4}{31104} - \frac{\pi^6}{33592320} \cdots$.

   (b) Using the geometric series $\sum_{n=0}^{\infty} (-1)^n x^{2n}$ $, -1 < x < 1$ and the information above, show that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$, $-1 \le x \le 1$. Use that to approximate π in two decimal places.

   (*Note*: Just use the first four terms of the power series)