

Quiz 6

Math 200-A

$$1. e^{3x} + 2x + 1 = 1 + 0 + 1 = 2$$
$$P_0 = 2$$

$$3e^{3x} + 2 = 3 + 2 = 5$$

$$P_1 = 2 + \frac{5(x-0)}{1!} = 2 + 5x$$

$$9e^{3x} = 9$$

$$P_2 = 2 + 5x + \frac{9x^2}{2!}$$

$$2. f(x) = 4x^3 - 3x^2 + 5x - 1 \quad f(2) = 29$$
$$f'(x) = 12x^2 - 6x + 5 \quad f'(2) = 41$$
$$f''(x) = 24x - 6 \quad f''(2) = 42$$
$$f'''(x) = 24 \quad f'''(2) = 24$$
$$f^{(4)}(x) = 0 \quad f^{(4)}(2) = 0$$

$$29 + 41(x-2) + \frac{42(x-2)^2}{2!} + \frac{24(x-2)^3}{3!}$$

After the third derivative they all are zero.

This is necessary because the series above exactly approximates the real function, which is of 3rd degree.

$$3. f(1) = 1$$
$$f'(1) = -2x^{-3} = -2$$
$$f''(1) = 6x^{-4} = 6$$

$$\sum_{n=0}^{\infty} f(x) = 1 - 2(x-1) + 3(x-1)^2 - \dots$$

$$\sum_{n=0}^{\infty} (-1)^n (n+1)(x-1)^n$$

$$4. \int_0^1 \int_{y^{1/2}}^{y^4} (x^{1/2} - y^2) dx dy = \int_0^1 \left(\frac{2}{3} x^{3/2} - y^2 x \right) \Big|_{y^{1/2}}^{y^4} dy$$

$$\int_0^1 \left(\frac{2}{3} y^{\frac{3}{4}} - y^{\frac{5}{2}} + \frac{1}{3} y^6 \right) dy = \left(\frac{8}{21} y^{\frac{7}{4}} - \frac{2}{7} y^{\frac{7}{2}} + \frac{1}{21} y^7 \right) \Big|_0^1 = \frac{8}{21} - \frac{2}{7} + \frac{1}{21} = \frac{1}{7}$$

$$5. \int_0^1 \int_x^{2-x} (x^2 + y^2) dy dx = \int_0^1 \left(x^2 y + \frac{y^3}{3} \right) \Big|_x^{2-x} dx =$$

$$\int_0^1 \left(x^3 + \frac{1}{3} x^3 - 2x^2 + x^3 - \frac{1}{3} (2-x)^3 \right) dx = \int_0^1 \left(\frac{8}{3} x^3 - 4x^2 + 4x - \frac{8}{3} \right) dx =$$

$$\left. \frac{8}{12} x^4 - \frac{4}{3} x^3 + \frac{4}{2} x^2 - \frac{8}{3} x \right|_0^1 = -\frac{2}{3} + \frac{4}{3} - 2 + \frac{8}{3} = \frac{4}{3}$$