Midterm Exam

Math 200-Section A
(Fall 2005)

Solve the following problems. Show all your work in the space under each problem.

1. Find the equation of the plane that passes through the point P = (1, -2, -1) and is perpendicular to the line of intersection of the planes $x + 2y - z = 2$ and $x - y + z = 1$. (2 pts)

2. Find the length of the curve $\mathbf{r}(t) = (2t)i + (2t)j + (3-t)k$ from (0, 0, 3) to (2, 2, 2). (*Note*: Make sure you write the correct limits during the calculation of the integral) (2 pts)

3. Given $\mathbf{r}(t) = (3\sin t)i + (3\cos t)j + (4t)k$, show that $4\kappa + 3\tau = 0$, where κ and τ are the curvature and torsion, respectively, of the curve $\mathbf{r}(t)$. (2 pts)
(*Hint*: Use the formulas $\kappa = \dfrac{|v \times a|}{|v|^3}$ and $\tau = \dfrac{\det(\dot{x}, \dot{y}, \dot{z}; \ddot{x}, \ddot{y}, \ddot{z}; \dddot{x}, \dddot{y}, \dddot{z})}{|v \times a|^2}$)

4. (a) Calculate the limit: $\lim_{(x,y)\to(0,\frac{\pi}{2})} \frac{xy-1+3\sin y}{\cos x+1}$ (4 pts)

(b) Let $f(x)=\frac{3xy^2}{x^3+y^3}$. Show that $f(x)$ has no limit as $(x, y) \to (0,0)$.

5. Find the directional derivative of $f(x, y, z) = 2xy - y^2 + z$ at P = (0, 1, 1) in the direction of $v = 2i + j - k$. (2 pts)

6. Find the equations for the <u>tangent plane</u> and <u>normal line</u> for $x^3 - xy - y^2 - xz = 0$ at P = (1, 2, -1). (2 pts)

7. Given $w = xy - z$, where $x = \cos t,\ y = \sin t,\ z = t$, find $\dfrac{dw}{dt}$ at $t = \dfrac{\pi}{4}$. (3 pts)

8. Find all the local maxima, local minima, and saddle points of the following function:

$$f(x, y) = 4xy - x^4 - y^4$$ (3 pts)