Midterm Exam

Math 258
(Fall 2005)

Solve the following problems. Show all your work in the space under each problem.

1. Construct a truth table for each of the compound propositions $\neg(p \oplus q)$ and $p \leftrightarrow q$ and determine whether they are equivalent. (2 pts)

2. Determine the truth value of the following statements, if the universe of discourse consists of all real numbers. Explain. (2 pts)

   (a) $\forall x \exists y (xy = 0)$

   (b) $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$

   (c) $\forall x \exists y (x + y = 1)$

   (d) $\exists x \exists y (x + y \neq y + x)$

3. Prove or disprove that product of a non-zero rational number and an irrational number is irrational. (*Hint*: Use the fact that the product of two rational numbers is rational) (2 pts)

4. (a) Find two sets A and B such that $A \in B$ and $A \subset B$. (2 pts)

   (b) Find the power set of the set $\{\varnothing, \{\varnothing, \{\varnothing\}\}\}$.

5. The **symmetric difference** of two sets A and B, denoted by $A \oplus B$, is the set containing those elements that belong to either A or B, but not in both. Prove or disprove: If $A \oplus C = B \oplus C$, then $A = B$. (2 pts)

6. Show that if $2^n - 1$ is a prime, then n is a prime. (2 pts)
   (*Hint*: Use the identity $2^{ab} - 1 = (2^a - 1)(2^{a(b-1)} + 2^{a(b-2)} + \cdots + 2^a + 1)$)

7. Find the gcd(224,106) using the Euclidean algorithm. (2 pts)

8. Use the *extended Euclidean algorithm* to write the gcd(218,105) as a linear combination of 218 and 105. Use that relation to find the inverse of 105 in $\mathbb{Z}_{218}$. (2 pts)

9. Solve the congruence $3x \equiv 7 \bmod 17$. List at least three integers that are solutions of the congruence. (2 pts)

10. Solve the system of congruences: (2 pts)

$$x \equiv 1 \bmod 2$$
$$x \equiv 2 \bmod 3$$
$$x \equiv 3 \bmod 5$$