

# QUIZ 1

(Math 258)

1. Determine whether each of the following arguments is valid: (4 pts)

(a) If  $n$  is a real number such that  $n > 1$ , then  $n^2 > 1$ .

Suppose that  $n^2 > 1$ . Then,  $n > 1$ .

(b) The number  $\pi$  is irrational if it is not the ration of two integers.

Therefore, since  $\pi$  cannot be written as in the form  $\frac{a}{b}$  where  $a$  and  $b$  are integers,  $\pi$  is irrational.

(c) If  $n$  is a real number such that  $n > 2$ , then  $n^2 > 4$ .

Suppose that  $n^2 \leq 4$ , then  $n \leq 2$ .

(d) If  $n$  is a real number such that  $n > 2$ , then  $n^2 > 4$ .

Suppose that  $n \leq 2$ , then  $n^2 \leq 4$ .

2. Prove that if  $3n + 2$  is even, where  $n$  is an integer, then  $n$  is even. (4 pts)  
(Hint: Use proof by contradiction)

3. Prove or disprove that the product of a non-zero rational number and an irrational number is irrational. (4 pts)

4. Prove that if  $x$  and  $y$  are real numbers, then  $\max(x, y) + \min(x, y) = x + y$ . (3 pts)  
(Hint: Use proof by cases)

5. Show that the following statements are equivalent: (i)  $x$  is rational (3 pts)  
(ii)  $\frac{x}{2}$  is rational  
(iii)  $3x - 1$  is rational.

6. **Wason's Cards:** Suppose you are given four cards, where each card has a letter (2 pts) on one side and a number on the other. The four cards read: A, D, 4 and 7. You are also given the following statement:

“ If a card has a vowel on one side, then that card has an even number on the other side ”

Which cards you have to turn over in order to check whether the statement above is false ? Explain.