Key

QUIZ 1

(Math 200-Section A)

1. Find the parametric equations of the line that passes through the point P = (2, 1, 0) and is perpendicular to the vectors $u = 2i + 3j + k$ and $v = i + j - 2k$. (4 pts)

The vector $w = \bar{u} \times \bar{v}$ is // to the line

$\bar{w} = \bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 1 & 1 & -2 \end{vmatrix} = -7i + 5j - k$ So, $x = 2 - 7t$, $y = 1 + 5t$, $z = -t$

2. Find the parametric equations of the line of intersection of the planes $3x - 2y - 2z = 3$ and $2x + y - 2z = 2$.

For a point P on the line: Set $x = 0$, so $\begin{cases} -2y - 2z = 3 \\ y - 2z = 2 \end{cases} \Rightarrow y = -1/3$, So, $z = -7/6$

So, $P = (0, -1/3, -7/6)$

For a vector w parallel to the line: $w = \bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} i & j & k \\ 3 & -2 & -2 \\ 2 & 1 & -2 \end{vmatrix} = 6i + 2j + 7k$

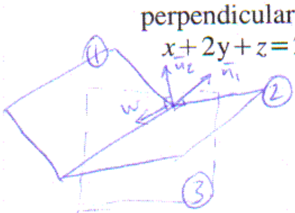
So, $x = 6t$, $y = -1/3 + 2t$, $z = -7/6 + 7t$

3. Find the point of intersection between the line $x = 1 - 2t$, $y = 1$, $z = 2t$ and the plane $2x - z = 14$.

Put ① into ②: $2(1 - 2t) - 2t = 14 \Rightarrow -6t = 12 \Rightarrow t = -2$

For $t = -2 \longrightarrow x = 5, y = 1, z = -4$. So, the point is $(5, 1, -4)$.

4. Find the equation of the plane that passes through the point P = (2, 1, -1) and is perpendicular to the line of intersection of the planes $2x + y - z = 3$ and $x + 2y + z = 2$.



The vector w is parallel to the line of intersection.

Hence, w ⊥ to the plane ③.

$w = \bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3i - 3j + 3k$

So, the equ of the plane is $3(x - 2) - 3(y - 1) + 3(z + 1) = 0$

$\Rightarrow 3x - 3y + 3z = 0$

5. Show that if the planes $A_1 x + B_1 y + C_1 z = D_1$ and $A_2 x + B_2 y + C_2 z = D_2$ are parallel, then $\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2} = \dfrac{C_1}{C_2}$.

Since ① // ② $\Rightarrow \bar{n}_1 \parallel \bar{n}_2 \Rightarrow \bar{n}_1 \times \bar{n}_2 = 0$

ie, $\begin{vmatrix} i & j & k \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} = 0 \Rightarrow (B_1 C_2 - B_2 C_1) i - (A_1 C_2 - C_1 A_2) j + (A_1 B_2 - A_2 B_1) k = 0$ $(= 0i + 0j + 0k)$

$\Rightarrow B_1 C_2 - B_2 C_1 = 0, \quad A_1 C_2 - C_1 A_2 = 0, \quad A_1 B_2 - A_2 B_1 = 0$

$\Rightarrow \dfrac{B_1}{B_2} = \dfrac{C_1}{C_2}, \quad \dfrac{A_1}{A_2} = \dfrac{C_1}{C_2}, \quad \dfrac{A_1}{A_2} = \dfrac{B_1}{B_2}$, ie $\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2} = \dfrac{C_1}{C_2}$