Key

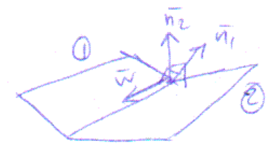
QUIZ 1

(Math 200-Section B)

1. Find the parametric equations of the line ℓ that passes through the point P = (3, 0, 2) and is perpendicular to the vectors $u = 2i + 3j + 1k$ and $v = 3i + 4j + 5k$.

A vector // to ℓ is $\bar{w} = \bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 3 & 4 & 5 \end{vmatrix} = 11i - 7j - k$

So, $\begin{cases} x = 3 + 11t \\ y = -7t \\ z = 2 - t \end{cases}$



2. Find the parametric equations of the line of intersection of the planes $3x - 6y - 2z = 3$ (1) and $2x + y - 2z = 2$ (2)

The vector $\bar{w} = \bar{n}_1 \times \bar{n}_2$ is // to the line of intersection

ie, $\bar{w} = \begin{vmatrix} i & j & k \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} = 14i + 2j + 15k$

A point P on the line is: Set $y = 0$, then $\left.\begin{matrix} 3x - 2z = 3 \\ 2x - 2z = 2 \end{matrix}\right\} \Rightarrow x = 1$

Hence $z = 0$

ie, $P = (1, 0, 0)$

So the eqn of the line is:

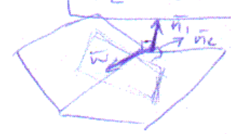
$\begin{cases} x = 1 + 14t \\ y = 2t \\ z = 15t \end{cases}$

3. Find the point of intersection between the line $x = -1 + 2t,\ y = -1,\ z = 3t$ (1) and the plane $2x - 3z = 8$ (2).

Put (1) into (2): $2(-1+2t) - 3(3t) = 8 \Rightarrow -5t = 10$
$\Rightarrow t = -2$

For $t = -2$: $x = -5,\ y = -1,\ z = -6 \Rightarrow P = (-5, -1, -6)$

4. Find the equation of the plane that passes through the point P = (2, 1, -1) and is perpendicular to the line of intersection of the planes $2x + y - z = 3$ (1) and $x + 2y + z = 2$ (2).



The vector $\bar{w} = \bar{n}_1 \times \bar{n}_2$ is $\perp$ to the plane we are looking for.

ie, $\bar{w} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3i + 3j + 3k$

So, the equ of the plane is: $3(x-2) + 3(y-1) + 3(z+1) = 0 \Rightarrow 3x + 3y + 3z = 0$

5. Show that if the planes $A_1x + B_1y + C_1z = D_1$ (1) and $A_2x + B_2y + C_2z = D_2$ (2) are parallel, then $\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2} = \dfrac{C_1}{C_2}$.

(1) // (2) $\Rightarrow \bar{n}_1 \times \bar{n}_2 = 0 \Rightarrow \begin{vmatrix} i & j & k \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} = 0$

$\Rightarrow (B_1C_2 - B_2C_1)i - (A_1C_2 - A_2C_1)j + (A_1B_2 - B_1A_2)k = 0$

$\Rightarrow \left.\begin{matrix} B_1C_2 - B_2C_1 = 0 \\ A_1C_2 - A_2C_1 = 0 \\ A_1B_2 - B_1A_2 = 0 \end{matrix}\right\}$

$\Rightarrow \begin{cases} \dfrac{B_1}{B_2} = \dfrac{C_1}{C_2} \\ \dfrac{A_1}{A_2} = \dfrac{C_1}{C_2} \end{cases} \Rightarrow \dfrac{B_1}{B_2} = \dfrac{C_1}{C_2}$