QUIZ 1

(Math 258)

1. Determine whether each of the following arguments is valid: (4 pts)

(a) If n is a real number such that $n > 1$, then $n^2 > 1$.
Suppose that $n^2 > 1$. Then, $n > 1$.

Answer: Invalid.

(b) The number π is irrational if it is not the ration of two integers.
Therefore, since π cannot be written as in the form $\frac{a}{b}$ where a and b are integers, π is irrational.

Answer: Valid.

(c) If n is a real number such that $n > 2$, then $n^2 > 4$.
Suppose that $n^2 < 4$, then $n < 2$.

Answer: Valid.

(d) If n is a real number such that $n > 2$, then $n^2 > 4$.
Suppose that $n < 2$, then $n^2 < 4$.

Answer: Invalid.

2. Prove that if $3n + 2$ is even, where n is an integer, then n is even. (4 pts)
(Hint: Use proof by contradiction)

Answer: *Suppose n is odd* $\Rightarrow n = 2k + 1$

$$
\begin{aligned}
\textit{Now},\ 3n+2 &= 3(2k+1)+2 \\
&= 6k+3+2 \\
&= 6k+5 \\
&= 6k+4+1 = 2(3k+2)+1 \\
&= 2m+1,\ \textit{ie},\ 3n+2 \textit{ is odd which contradicts } 3n+2 \textit{ being even.}
\end{aligned}
$$

3. Prove or disprove that the product of a non-zero rational number and an irrational number is irrational. (4 pts)

Answer : Let $\frac{a}{b}$ *be a rational number ,* $a, b \in \mathbb{Z}$.

Let i be an irrational number.

$\frac{a}{b} \cdot i = \frac{ai}{b}$

But $a \cdot i$ *is not an integer , so*

$\frac{a}{b} \cdot i$ *is not rational*

4. Prove that if x and y are real numbers, then max(x, y) + min(x, y) = $x + y$. (3 pts)

(Hint: Use proof by cases)

***Answer:* $x > y$:**

$max(x, y) + min(x, y) = x + y$ *because max returns* x, *min returns* y.

$x < y$:

$max(x, y) + min(x, y) = y + x$ *because max returns* y, *min returns* $x = x + y$.

$x = y$:

$max(x, y) + min(x, y) = x + x$ **or** $y + y = x + y$ *because* $x = y$.

5. Show that the following statements are equivalent: (i) x is rational (3 pts)

(ii) $\frac{x}{2}$ is rational

(iii) $3x - 1$ is rational.

Answer:

$(i) = (ii) : x \text{ is rational} \Rightarrow x = \frac{a}{b}. \text{ So } \frac{x}{2} = \frac{a}{2b}, \text{ ie. rational.}$

$(ii) = (iii) : \frac{x}{2} \text{ is rational} \Rightarrow \frac{x}{2} = \frac{a}{b} \Rightarrow x = \frac{2a}{b}, \text{ ie. rational.}$

$$3x - 1 = 3\left(\frac{2a}{b}\right) - 1 = \frac{6a}{b} - 1 = \frac{6a - b}{b} = \frac{p}{q}, \text{ ie.rational.}$$

$(iii) = (i) : 3x - 1 \text{ is rational} \Rightarrow 3x - 1 = \frac{a}{b}$

$$\Rightarrow 3x = \frac{a}{b} - 1$$

$$\Rightarrow 3x = \frac{a - b}{b} \Rightarrow x = \frac{a - b}{3b} = \frac{p}{q}, \text{ ie. rational.}$$

6. ***Wason 's Cards:*** Suppose you are given four cards, where each card has a letter (2 pts) on one side and a number on the other. The four cards read: A, D, 4 and 7. You are also given the following statement:

" If a card has a vowel on one side, then that card has an even number on the other side "

Which cards you have to turn over in order to check whether the statement above is <u>false</u> ? Explain.

Answer: You have to turn over A and 7. The statement above is of the form p → q.
So the two ways to falsify it is to have :
(i): p true and q false
ie, Choose A and turn it to see if get an odd number.
or (ii): q true and p false
ie, Choose 7 and turn it to see if you get a consonant.