Homework #2
Math 200-A

1. $\lim_{r->0} \frac{(r\cos\theta)^3 - (r\cos\theta)\ (r\sin\theta)^2}{(r\cos\theta)^2 + (r\cos\theta)^2} = \lim_{r->0} \frac{r^3\cos^3\theta - r\cos\theta(r^2\sin^2\theta)}{r^2\cos^2\theta + r^2\sin^2\theta} =$

$\lim_{r->0} \frac{r^2(r\cos^3\theta - r\cos\theta\sin^2\theta)}{r^2(\cos^2\theta - \sin^2\theta)} = \lim_{r->0} \frac{r\cos^3\theta - r\cos\theta\sin^2\theta}{1} = \frac{0}{1} = 0$

2. $f(x) = 2z^3 - 3(x^2 + y^2)z = 2z^3 - 3x^2z - 3y^2z$

$(\partial f/\partial x) = -6xz$ $(\partial f/\partial y) = -6yz$ $(\partial f/\partial z) = 6z^2 - 3x^2 - 3y^2$

$(\partial^2 f/\partial x^2) = -6z$ $(\partial^2 f/\partial y^2) = -6z$ $(\partial^2 f/\partial z^2) = 12z$

$(\partial^2 f/\partial x^2) + (\partial^2 f/\partial y^2) + (\partial^2 f/\partial z^2) = -6z - 6z + 12z = 0$

3. $(\partial w/\partial u) = (\partial w/\partial x) * (\partial x/\partial u) + (\partial w/\partial y) * (\partial y/\partial u) + (\partial w/\partial z) * (\partial z/\partial u)$

$= (y)(-v^2/u^2) + (x)(1) + (1/2)(-\sin u)$

$= (u + v)(-v^2/u^2) + (v^2/u^2) + (1/\cos u)(-\sin u)$

$= (-1 + 2)(-4) + (4) - \tan(-1)$

$= 4 - 8 - 4 - \tan(-1)$

$= -8 - \tan(-1)$

$(\partial w/\partial v) = (\partial w/\partial x) * (\partial x/\partial v) + (\partial w/\partial y) * (\partial y/\partial v) + (\partial w/\partial z) * (\partial z/\partial v)$

$= (y)(2v/u) + (x) + (1/z)(0)$

$= (u + v)(2v^2/u) + (v^2/u)$

$= (-1 + 2)(-4) - 4$

$= -8$

4. $f(x) = x^2 - 3xy + 4y^2$

$f_x(1,2) = (2x - 3y + 0) = 2 - 6 = -4$

$f_y(1,2) = (0 - 3x + 8y) = -3 + 16 = 13$

$\nabla f = -4i + 13j$

$|\nabla f| = 13.6$

Since the magnitude of the ∇ f is the direction f increases most rapidly. So it can not = = 14.

5. $\nabla f * \mathbf{u} = \sqrt{3}$
$\mathbf{u} = (1/\sqrt{3})i - (1/\sqrt{3})j + (1/\sqrt{3})k$
$\nabla f = i - j + k$

6. $\nabla f = f_x i + f_y j + f_z k$
$D_i f = \nabla f * i = (f_x i + f_y j + f_z k)(i) = f_x$
$D_j f = \nabla f * j = (f_x i + f_y j + f_z k)(j) = f_y$
$D_k f = \nabla f * k = (f_x i + f_y j + f_z k)(k) = f_z$

7. $f(x) = x^3 + 3xy + y^3$
$f_x = 3x^2 + 3y$ $f_{xx} = 6x$ $f_{xy} = 3$
$f_y = 3x + 3y^2$ $f_{yy} = 6y$

$3x^2 + 3y = 0$ at (0, 0), (-1,-1), (-1, 1)
$3x + 3y^2 = 0$ at (0, 0), (-1,-1), (1,-1)

So local extreme are (0, 0) and (-1,-1)

f has a local maximum if ($f_{xx} < 0$) and ($f_{xx} f_{yy} - f_{xy}^2 > 0$) at (x, y).
We will check if (-1, -1) is a local maximum.

- First we notice that ($f_{xx} < 0$) since (6x) = -6.
- So the first part is satisfied.
- Second we see that ($f_{xx} f_{yy} - f_{xy}^2 > 0$) at (-1, -1).
- $(6x)(6y) - 3^2 = (-6)(-6) - 9 = 27$.
- So (-1, -1) is a local maximum.

f has a saddle point if ($f_{xx} f_{yy} - f_{xy}^2 < 0$) at (x, y).
We will check if (0,0) is a saddle point.

- First we notice that ($f_{xx} f_{yy} - f_{xy}^2 < 0$) at (0, 0).
- $(6x)(6y) - 3^2 = (0)(0) - 9 = -9$.
- So (0,0) is a saddle point.