QUIZ 3

(Math 200-Section A)

1. Find the limits: (a) $\lim_{(x,y)\to(\frac{\pi}{2},0)} \frac{\cos y - 2}{y - \sin x}$ (b) $\lim_{(x,y)\to(1,1)} \frac{xy - y - 2x + 2}{x - 1}$ (4 pts)

(a). $\lim_{(x,y)\to(\frac{\pi}{2},0)} \frac{\cos y - 2}{y - \sin x} = \frac{\cos(0) - 2}{0 - \sin(\pi/2)} = \frac{1-2}{0-1} = 1$

(b). $\lim_{(x,y)\to(1,1)} \frac{xy - y - 2x + 2}{x - 1} = \frac{y(x-1) - 2(x-1)}{(x - 1)} = \frac{y-2}{1} = 1 - 2 =$ -1

2. Check whether the function $f(x,y) = \frac{1}{x^2 - y}$ is continuous at (-1,1). Can you describe all points of discontinuity of f ? (4 pts)

$f(x,y) = \frac{1}{x^2 - y} = \frac{1}{1-1} = \frac{1}{0}$

Since f(x, y) is not defined at (-1, 1) it is not continuous there.

f is discontinuous at all point where $x^2 =$ y.

3. By considering different paths of approach, show that the function $f(x,y)=\frac{xy}{|xy|}$ has no limit as $(x,y)\to(0,0)$. (4 pts)

First take the limit as $(x, y) \to (0^+, 0^+)$. Which is 1.

Then take the limit as $(x, y) \to (0^-, 0^+)$. Which is -1.

Since there are two different limits as f approaches (0,0) the limit does not exists.

4. Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of the function $f(x,y)=e^{-x}\sin(x+y)$.

(4 pts)

$$\frac{\partial f}{\partial x} = e^{-x}\cos(x + y) - e^{-x}\sin(x + y)$$

$$\frac{\partial f}{\partial y} = e^{-x}\cos(x + y)$$

5. Find the value of $\frac{\partial z}{\partial x}$ at (1,1,1) if the equation $xy+z^3x-2yz=0$ defines z as a function of the two independent variables x and y, and the partial derivatives exist.

(4 pts)

$$y + z^3 + \frac{\partial z}{\partial x}3z^2x - \frac{\partial z}{\partial x}2y = 0$$

$$\frac{\partial z}{\partial x}(3z^2x - 2y) = -y - z^3$$

$$\frac{\partial z}{\partial x} = \frac{-y - z^3}{3z^2x - 2y} = \frac{-1 - 1}{3 - 2} = -2$$