Quiz 4

(Math 200-Section A)

1. Find the directional derivative of $f(x, y) = 2xy - 3y^2$ at $P = (5,5)$ in the direction of $v = 4i + 3j$. (4 pts)

   $|\mathbf{v}| = 5$ $(\mathbf{v}/|\mathbf{v}|) = \mathbf{u} = 4/5\mathbf{i} + 3/5\mathbf{j}$

   $f_x(5,5) = (2y - 0) = 10$
   $f_y(5,5) = (2x - 6y) = -20$

   Gradient of $f = f_x + f_y = 10i - 20j$

   $(D_u f)|_{(5,5)} = (10i - 20j)*(\mathbf{u}) = 8 - 12 = -4$

2. Find the directions in which $f(x,y) = x^2 - xy + y^2$: (4 pts)

   a) Increases most rapidly at $P = (1,-1)$

   b) Decreases most rapidly at $P = (1,-1)$

   c) Has zero change at $P = (1,-1)$

   $f_x(1,-1) = (2x - y + 0) = 3$
   $f_y(1, -1) = (0 - x + 2y) = -3$

   Gradient of $f = 3i - 3j$

   (a) |Gradient of f| $= (3 / \sqrt{18})i - (3 / \sqrt{18})j$
   (b) -|Gradient of f| $= -(3 / \sqrt{18})i + (3 / \sqrt{18})j$
   (c) Any direction orthogonal to the gradient is a direction of zero change.
   $(3 / \sqrt{18})i + (3 / \sqrt{18})j$ or $-(3 / \sqrt{18})i - (3 / \sqrt{18})j$

3. Find the equations for the <u>tangent plane</u> and <u>normal line</u> for $x^2 - xy - y^2 - z = 0$ at $P = (1,1,-1)$. (4 pts)

$f_x\,(1,1,-1) = (2x - y - 0 - 0) = (2 - 1) = 1$
$f_y\,(1,1,-1) = (0 - x - 2y - 0) = (-1 - 2) = -3$
$f_z\,(1,1,-1) = (0 - 0 - 0 - 1) = -1$

Gradient of f = i – 3j –k

plane = (x – 1) – 3(y – 1) – (z + 1) = 0 or x – 3y – z = -1

line = x = 1 + t
y = 1 – 3t
z = -1 - t

4. Find the equation for the <u>tangent line</u> for $x^2 - y = 1$ at $P = (\sqrt{2},1)$. (4 pts)

$f_x\,(\sqrt{2},1) = (2x - 0) = 2\sqrt{2}$
$f_y\,(\sqrt{2}, 1) = (0 - 1) = -1$
Gradient of f = $2\sqrt{2}i - j$

$2\sqrt{2}(x - \sqrt{2}) - (y + 1) = 0$

$2\sqrt{2}x - 4 - y + 1 = 0$

$2\sqrt{2}x - y = 3$

$y = 2\sqrt{2}x - 3$

5. Find the parametric equations of the line tangent to the curve of intersection of the surfaces $xyz = 1$ and $x^2 + 2y^2 + 3z^2 = 6$ at $P = (1,1,1)$. (4 pts)

Let $f(x) = xyz$ and let $g(x) = x^2 + 2y^2 + 3z^2$

$f_x\,(1,1,1) = yz = 1$
$f_y\,(1,1,1) = xz = 1$
$f_z\,(1,1,1) = xy = 1$

$g_x\,(1,1,1) = 2x + 0 + 0 = 2$
$g_y\,(1,1,1) = 0 + 4y + 0 = 4$
$g_z\,(1,1,1) = 0 + 0 + 6z = 6$

u = Gradient of f = i + j + k
v = Gradient of g = 2i + 4j + 6k
u x **v** = 2i – 4j + 2k
Tangent Line = x = 1 + 2t, y = 1 – 4t, z = 1 + 2t