Key

QUIZ 4

(Math 200-Section B)

1. Find the directional derivative of $f(x,y) = 2x^2 - 3xy$ at P = (2,3) in the direction of $v = 3i - 4j$. (4 pts)

$\nabla f(x,y) = (4x-3y)\hat{\imath} + (-3x)\hat{\jmath}$

let $\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{\jmath}$

$\left(\frac{\partial f}{\partial s}\right)_{\vec{u},P} = \nabla f(2,3)\cdot\vec{u} = (-\hat{\imath} - 6\hat{\jmath})\cdot\left(\frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{\jmath}\right)$

$= -\frac{3}{5} + \frac{24}{5}$

$= \boxed{\frac{21}{5}}$

2. Find the directions in which $f(x,y) = x^2 + xy + y^2$: (4 pts)

(a) increases most rapidly at P = (-1,1)

(b) decreases most rapidly at P = (-1,1)

(c) has zero change at P = (-1,1)

$\nabla f(x,y) = (2x+y)\hat{\imath} + (x+2y)\hat{\jmath}$

(a) $\nabla f(-1,1) = (-2+1)\hat{\imath} + (-1+2)\hat{\jmath} = -\hat{\imath} + \hat{\jmath}$. direction: $\boxed{-\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}}$

(b) $-\nabla f(-1,1) = \hat{\imath} - \hat{\jmath}$. direction: $\boxed{\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}}$

(c) perpendicular directions $\boxed{\frac{-1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath} \text{ and } \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}}$

3. Find the equations for the tangent plane and normal line for $x^2 - xy - y^2 - z = 0$ at P = (1,1,-1). (4 pts)

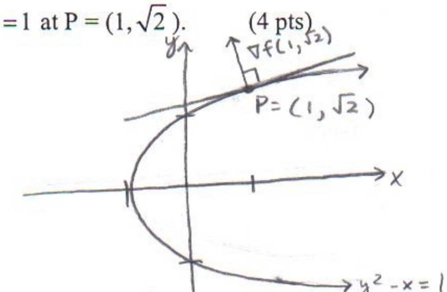
Let $f(x,y,z) = x^2 - xy - y^2 - z$; then $\nabla f(x,y,z) = (2x-y)\hat{\imath} + (-x-2y)\hat{\jmath} - \hat{k}$, and $\nabla f(1,1,-1) = (2-1)\hat{\imath} + (-1-2)\hat{\jmath} - \hat{k} = \underline{\hat{\imath} - 3\hat{\jmath} - \hat{k}}$. This vector is normal to the surface $f(x,y,z) = c$ at point $(1,1,-1) = P$.

The tangent plane is therefore $(1,-3,-1) \cdot (\vec{r} - (1,1,-1)) = 0$,

i.e. $\boxed{1(x-1) - 3(y-1) - 1(z+1) = 0}$ or $x - 3y - z + 1 = 0$,

and the normal line is $\vec{r}(t) = (1,1,-1) + t(1,-3,-1)$, i.e. $\boxed{\begin{cases} x = 1+t \\ y = 1-3t \\ z = -1-t \end{cases}}$.

4. Find the equation for the tangent line for $y^2 - x = 1$ at P = $(1, \sqrt{2})$. (4 pts)



Let $f(x,y) = y^2 - x$

then $\nabla f(x,y) = (-1)\hat{\imath} + (2y)\hat{\jmath}$

and $\nabla f(1,\sqrt{2}) = \underline{-\hat{\imath} + 2\sqrt{2}\hat{\jmath}}$. This vector is normal to the surface $f(x,y) = c$ at point $(1,\sqrt{2}) = P$.

The tangent line is therefor $(-1, 2\sqrt{2}) \cdot (\vec{r} - (1,\sqrt{2})) = 0$,

i.e. $(-1)(x-1) + (2\sqrt{2})(y - \sqrt{2}) = 0 \Rightarrow \boxed{-x + 2\sqrt{2}y - 3 = 0}$

or $y = \frac{1}{2\sqrt{2}}x + \frac{3}{2\sqrt{2}}$.

5. Find the parametric equations of the line tangent to the curve of intersection of the surfaces $xyz = 1$ and $x^2 + 2y^2 + 3z^2 = 6$ at P = (1,1,1). (4 pts)

(Tangent line is perpendicular to the normals of both surfaces at the point, P)

Let $f(x,y,z) = xyz$ and $g(x,y,z) = x^2 + 2y^2 + 3z^2$

$\nabla f(x,y,z) = (yz, xz, xy)$ $\quad \nabla g(x,y,z) = (2x, 4y, 6z)$

$\nabla f(1,1,1) = (1,1,1)$ $\quad \nabla g(1,1,1) = (2,4,6)$ ← These vectors are normal to the surfaces at $(1,1,1) = P$.

The tangent line is therefore parallel to $\nabla f(1,1,1) \times \nabla g(1,1,1) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 4 & 6 \end{vmatrix} = (2,-4,2)$.

So the line is $\vec{r}(t) = (1,1,1) + t(2,-4,2)$, i.e. $\boxed{\begin{cases} x = 1+2t \\ y = 1-4t \\ z = 1+2t \end{cases}}$