

## Quiz 4

(Math 258)

1. Find the values of each of the following quantities:

a.  $P(5, 3)$

$$P(n, r) = n!/(n - r)!$$

$$P(5, 3) = 5!/(5-3)!$$

$$= 5!/2!$$

$$= (5 * 4 * 3 * 2 * 1) / (2 * 1)$$

$$= 60$$

b.  $C(6, 4)$

$$C(n, r) = n!/[r!(n - r)!]$$

$$C(6, 4) = 6!/[4!(6-4)!]$$

$$= 6!/(2! 4!)$$

$$= (6*5*4*3*2*1) / [(4*3*2*1)(2*1)]$$

$$= 15$$

2. How many ways are there for 8 men and 5 women to stand in line so that no two women stand next to each other? (Hint: Position the men first, and then consider the possible positions for women)

First position the men relative to each other. Since there are 8 men, there are  $P(8, 8)$  ways to do this. This creates nine slots where a woman (but not more than one woman) may stand: in front of the first man, between first and second, ...after the last man. We need to choose 5 of these positions, in order, for the first through fifth woman to occupy. This can be done in  $P(9, 5)$  ways. Therefore, there are  $P(8, 8) * P(9, 5)$  ways.

3. Use the Binomial Theorem to find the expansion of  $(x + y)^5$ .

$$\begin{aligned}(x + y)^5 &= \binom{5}{0}x^5y^0 + \binom{5}{1}x^4y^1 + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}x^1y^4 + \binom{5}{5}x^0y^5 \\ &= 1 \cdot x^5y^0 + 5 \cdot x^4y^1 + 10 \cdot x^3y^2 + 10 \cdot x^2y^3 + 5 \cdot x^1y^4 + 1 \cdot x^0y^5\end{aligned}$$

4. What is the coefficient of  $x^6$  in  $(1 + x)^7$ , without doing the expansion?

The coefficient of  $x^6$  is  $\binom{7}{6}$ .

5. When a single card is drawn from a 52-card, what is the probability of drawing a card other than a king?

|cards that aren't kings|

$$\text{-----} = 48/52 = 12/13$$

|total number of cards|

6. If a number is selected randomly from the set  $S = \{1, 2, \dots, 10\}$ , find the probability that the number will be odd or a multiple of 3.

**1 2 3 4 5 6 7 8 9 10**

5 are odd.

**1 2 3 4 5 6 7 8 9 10**

3 are divisible by three.

**1 2 3 4 5 6 7 8 9 10**

6 are odd or divisible by three.

Since there are 10 numbers, the probability of picking a number that is odd or a multiple of three is  $6/10$ , or  $3/5$ .

7. What is the probability to roll a 2 in a 3-number biased dice, in which 1 is three times as likely to come up as 3 and 2 is two times as likely to come up as 3?

**1 1 1 2 2 3**

There are 6 possibilities. The probability of rolling a 2 is  $2/6$ , or  $1/3$ .

Or:

$P(1) = 3 P(3)$ ,  $P(1) = 3 P(3)$  and  $P(1) + P(2) + P(3) = 1$ . From this system we get  $P(2) = 1/3$ .

8. Each year, John adds to his book collection a number of new publications that he believes will be of great interest. He has categorized each of his 20-year acquisitions as hardcover or paperback, and as fiction and non-fiction. The number of books in the various categories are shown in the following table:

<b>BOOKS</b>	Fiction (F)	Non-Fiction (N)	Totals
Hardcover (H)	3	5	8
Paperback (P)	8	4	12
Totals	11	9	20

If John chooses randomly one of these 20 books for this evening's reading, find the probability that the book will be:

- a. Hardcover

There are 8 hardcover books and 20 total books.

$8/20$

- b. Fiction, given that it is hardcover

There are 3 fiction books that are hardcover and 8 hardcover books.

$3/8$

- c. Hardcover and fiction.

There are 3 hardcover fiction books and 20 total books.

$3/20$