Quiz # 5 – Section A

1. First split up $\frac{4}{(4n-3)(4n+1)}$ using partial fractions.

$$\left[\frac{4}{(4n-3)(4n+1)} = \frac{A}{(4n-3)} + \frac{B}{(4n+1)}\right](4n-3)(4n+1)$$

4 = A(4n + 1) + B(4n – 3)
4 = A4n + A + B4n – 3B

4 = A – 3B
0 = A4n + B4n

Solve for A: A = 4 + 3B, substitute into the other equation.
0 = (4 + 3B)4n + B4n
0 = 16n + 16Bn
B = $\frac{-16n}{16n}$ = -1

So B = -1 and A = 1.

$$\sum_{n=1}^{\infty} \frac{1}{(4n-3)} - \frac{1}{(4n+1)}$$

$$\sum_{n=1}^{\infty} \frac{1}{(4n-3)} = 1 + 1/5 + 1/9 + \ldots..$$

$$\sum_{n=1}^{\infty} \frac{-1}{(4n+1)} = -1/5 -1/9 - 1/12 -\ldots\ldots\ldots$$

So the sum of the series is 1.

2. a) 1, $\frac{1}{e^3}$, $\frac{1}{e^6}$ b) r = $\frac{1}{e^3}$ c) $\frac{1}{1-e^{-3}}$

3. $\lim_{a \to \infty} \int_1^a \frac{3n}{\frac{3}{2}n^2 - 1} = \lim_{a \to \infty} \ln(\frac{3}{2}n^2 - 1)\Big|_1^a = \lim_{a \to \infty} \ln(\frac{3}{2}\text{n}^2 - 1) - \ln(.5) = \infty$

So it diverges.

4. $\frac{1+\sin n}{n^2} < \frac{1+1}{n^2}$ (because sin n <= 1)

$$\sum_{n=1}^{\infty} \frac{1+\sin n}{n^2} < \sum_{n=1}^{\infty} \frac{2}{n^2} = 2\sum_{n=1}^{\infty} \frac{1}{n^2}$$

Since we know $2\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, $\sum_{n=1}^{\infty} \frac{1+\sin n}{n^2}$ must also converge since it is less than $2\sum_{n=1}^{\infty} \frac{1}{n^2}$.

5. The ratio rule is $\lim_{n \to \infty} \frac{A_{n+1}}{A_n}$

$$\lim_{n \to \infty} \left(\frac{n!}{(n+1)(n+2)} \right) \left(\frac{(n+2)(n+3)}{(n+1)!} \right) = \lim_{n \to \infty} \frac{(n+3)}{(n+1)(n+1)} = \lim_{n \to \infty} \frac{(n+3)}{n^2 + 2n + 1} = 0$$

So it converges.

6. $\lim_{n \to \infty} \sqrt[n]{\frac{n^n}{(2^n)^2}} = \lim_{n \to \infty} \frac{\sqrt[n]{n^n}}{\sqrt[n]{2^{2n}}} = \lim_{n \to \infty} \frac{n}{4} = \infty$ So it diverges.