QUIZ 5

(Math 200-Section B)

1. Use partial fractions to find the sum of the series $\sum_{n=1}^{\infty}\frac{3}{(3n-2)(3n+1)}$. (4 pts)

Let $\frac{3}{(3n-2)(3n+1)}=\frac{A}{(3n-2)}+\frac{B}{(3n+1)}$

$\Rightarrow 3+0n = A(3n+1)+B(3n-2)$

So let $\begin{cases}3=A-2B\\0=3A+3B\end{cases}\Rightarrow\begin{cases}3=A-2B\\B=-A\end{cases}\Rightarrow\begin{cases}3=3A\\B=-A\end{cases}\Rightarrow\begin{cases}A=1\\B=-1\end{cases}$

So $\frac{3}{(3n-2)(3n+1)}=\frac{1}{(3n-2)}-\frac{1}{(3n+1)}$,

and
$$\begin{aligned}\sum_{n=1}^{\infty}\frac{3}{(3n-2)(3n+1)}&=\sum_{n=1}^{\infty}\left(\frac{1}{(3n-2)}-\frac{1}{(3n+1)}\right)\\&=\sum_{n=1}^{\infty}\left(\frac{1}{3n-2}\right)-\sum_{n=1}^{\infty}\left(\frac{1}{3n+1}\right)\\&=1+\sum_{n=2}^{\infty}\left(\frac{1}{3n-2}\right)-\sum_{n=1}^{\infty}\left(\frac{1}{3n+1}\right)\\&=1+\sum_{n=1}^{\infty}\left(\frac{1}{3(n+1)-2}\right)-\sum_{n=1}^{\infty}\left(\frac{1}{3n+1}\right)\\&=1+\sum_{n=1}^{\infty}\left(\frac{1}{3n+1}\right)-\sum_{n=1}^{\infty}\left(\frac{1}{3n+1}\right)\\&=1\end{aligned}$$

2. For the geometric series $\sum_{n=0}^{\infty}e^{-3n}$, find the following: (3 pts)

(a) the first three terms (b) The ratio r (c) The sum of the series.

(a) $1, e^{-3}, e^{-6}$

(b) $r=e^{-3}$

(c) sum $=\frac{a}{(1-r)}=\frac{1}{1-e^{-3}}$

3. Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty} \frac{5n}{\frac{5}{2}n^2 - 1}$ converges or diverges. (4 pts)

Let $f(x) = \dfrac{5x}{\frac{5}{2}x^2 - 1}$, a positive, decreasing function of x for all $x \geq 1$.

$$\int_1^{\infty} f(x)dx = \lim_{b\to\infty} \int_1^b \frac{5x}{\frac{5}{2}x^2 - 1} dx$$

$$= \lim_{b\to\infty} \ln\left|\frac{5}{2}x^2 - 1\right|\Bigg|_1^b$$

$$= \lim_{b\to\infty} \left(\ln\left|\frac{5}{2}b^2 - 1\right| - \ln\left|\frac{5}{2} - 1\right| \right)$$

$$= \lim_{b\to\infty} \ln\left|\frac{5}{2}b^2 - 1\right| - \ln(3/2)$$

diverges.

Since $a_n = \dfrac{5n}{\frac{5}{2}n^2 - 1} = f(n)$ for all $n \geq 1$, the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{5n}{\frac{5}{2}n^2 - 1}$ also diverges.

4. Use the Comparison Test to determine whether the series $\sum_{n=1}^{\infty} \frac{1 + \cos n}{n^2}$ converges or diverges. (3 pts)

Since cosine oscillates between –1 and 1, the numerator will oscillate between 0 and 2. We can see therefore see that the series has no negative terms, and that $\frac{1 + \cos n}{n^2} \leq \frac{2}{n^2}$ for all n. Therefore, since the series $\sum_{n=1}^{\infty} \frac{2}{n^2}$ converges (it is a p-series), the series $\sum_{n=1}^{\infty} \frac{1 + \cos n}{n^2}$ also converges.

5. Use the Ratio Test to determine whether the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$ converges or diverges.

(3 pts)

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{e^{n+1}}{(n+1)!}}{\frac{e^n}{n!}} = \lim_{n\to\infty} \frac{e^1}{(n+1)} = 0 < 1, \text{ so the series converges.}$$

6. Use the n-th Root Test to determine whether the series $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^n}$ converges or diverges.

(3 pts)

$$\rho = \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \left(\frac{(\ln n)^n}{n^n} \right)^{1/n} = \lim_{n\to\infty} \left(\frac{\ln n}{n} \right).$$

Using L'Hôpital's Rule, we get

$$\rho = \lim_{n\to\infty} \left(\frac{\ln n}{n} \right) = \lim_{n\to\infty} \left(\frac{1/n}{1} \right) = \lim_{n\to\infty} \left(\frac{1}{n} \right) = 0 < 1, \text{ so the series converges.}$$