Key

Final Exam

Math 258
(Fall 2005)

Solve the following problems. Show all your work in the space under each problem.

1. How many subsets of an 8-element set have more than three elements? (2 pts)
(*Hint*: It would be easier to calculate first those with three or fewer elements)

The 8-element set has $2^8 = 256$ subsets.

The number of subsets that contain 0 elements is $C(8,0) = \frac{8!}{0!8!} = 1$

" " " " " 1 " " $C(8,1) = \frac{8!}{1!7!} = 8$

" " " " " 2 " " $C(8,2) = \frac{8!}{2!6!} = 28$

" " " " " 3 " " $C(8,3) = \frac{8!}{3!5!} = 56$

So, the number of subset with ≤ 3 elments $= 1+8+28+56 = 93$

Hence, the number of subsets with >3 elements $= 256-93 = \boxed{163}$

2. What is the coefficient of x^4y^3 in $(2x+y)^7$, without doing the expansion? (2 pts)

$$\binom{7}{3}2^4 1^3 = \frac{7!}{3!4!}16 = \frac{\cancel{1.2.3}\ldots 7}{\cancel{1.2.3}\ 1.2.3.4}16 = \boxed{560}$$

3. Given a family with two children, what is the probability that both children are girls if the older child is a girl? (2 pts)

All possible cases are $S = \{gg, gb, bg, bb\}$

Since the older child is g, though, the sample space S reduces to $S' = \{gg, gb\}$

The favorable cases are $G = \{gg\}$

So, $P(gg) = |G|/|S'| = \boxed{1/2}$

4. Determine whether the relation $R = \{(x, y) \in Z \times Z \mid x = y^2\}$ is: (3 pts)

(a) reflexive

$\boxed{\text{No}}$

$(x,x) \notin R, \forall x$

eg For $x=2$:

$(2,2) \notin R$, bec. $2 \neq 2^2$

(b) symmetric

$\boxed{\text{No}}$

$(x,y) \in R \not\Rightarrow (y,x) \in R$

eg For $x=4$:

$(4,2) \in R \not\Rightarrow (2,4) \in R$

bec. $2 \neq 4^2$

(c) antisymmetric

$\boxed{\text{Yes}}$

$(x,y) \in R \wedge (y,x) \in R \Rightarrow x = y$,

bec. $x = y^2 \wedge y = x^2 \Rightarrow x = x^4$

$\Rightarrow x(1-x^3)=0$

$\Rightarrow x=0, x=1$

But for $x=0, x=1 \Rightarrow y=0, y=1$

ie, $x=y$.

5. Given the relations: $R_1 = \{(x,y) \in R \times R \mid x > y\}$, $R_2 = \{(x,y) \in R \times R \mid x \geq y\}$ and $R_3 = \{(x,y) \in R \times R \mid x = y\}$, find the following: (4 pts)

(a) $R_1 \cup R_2$

$R_1 \cup R_2 = \{(x,y) \mid x > y \text{ or } x \geq y\}$
$= \{(x,y) \mid x \geq y\}$
$= R_2$

(b) $R_1 \cap R_3$

$R_1 \cap R_3 = \{(x,y) \mid x > y \text{ and } x = y\}$
$= \emptyset$

(c) $R_2 - R_3$

$R_2 - R_3 = \{(x,y) / x \geq y \text{ and } x \neq y\}$
$= \{(x,y) / x > y\}$
$= R_1$

(d) $R_1 \circ R_3$

$R_1 \circ R_3 = \{(x,y) / x > a \text{ and } a = y\}$
$= \{(x,y) \mid x > y\}$
$= R_1$

6. A relation R on a set A is called **irreflexive** if ($x \in A \Rightarrow (x,x) \notin A, \forall x \in A$). In other words, no element in A relates to itself. Suppose now that a relation R is irreflexive. Is R^2 necessarily irreflexive? Explain. (2 pts)

$\boxed{\text{No}}$ eg. $R = \{(1,2),(2,1)\}$ on $A = \{1,2\}$ is irreflexive
but $R^2 = \{(1,1),(2,2)\}$ is not irreflexive.

7. Determine whether the following relation on the set of all functions from Z to Z is an equivalence relation. Check all the relevant properties: (3 pts)

$$R = \{(f,g) \mid f(0) = g(0) \text{ or } f(1) = g(1)\}$$

(a) $(f,f) \in R$, bec. $f(0) = f(0)$ or $f(1) = g(1)$, ie reflexive

(b) $(f,g) \in R \Rightarrow (g,f) \in R$, bec. $f(0) = g(0)$ or $f(1) = g(1) \Rightarrow g(0) = f(0)$ or $g(1) = f(1)$.
ie symmetric

(c) $(f,g) \in R, (g,h) \in R \not\Rightarrow (f,h) \in R$, bec. If $f(x) = 0$, $g(x) = x$, $h(x) = 1$
$\Rightarrow f(0) = g(0)$, $g(1) = h(1)$, but $f(0) \neq h(0)$ and $f(1) \neq h(1)$
ie, not transitive

Hence R is not an equivalence relation.

8. Find the equivalence relation induced by the following partition of the set S = {0,1,2,3,4,5}: (2 pts)

{0}, {1,2}, {3,4,5} .

$R = \{(0,0),(1,2),(2,1),(1,1),(2,2),(3,4),(4,3),(3,3),(4,4),(3,5),(5,3),$
$(5,5),(4,5),(5,4)\}$

Are elements 2 and 3 equivalent in this relation?

$\boxed{\text{No}}$, bec $(2,3) \notin R$