# Final Exam

**(Math 200, Fall 06)**

**Solve the following problems. Show all your work in the space under each problem.**

1. Use partial fractions to find the sum of the series $\sum_{k=3}^{\infty} \frac{1}{k^2 - k}$ . (10 pts)

2. True or False: If $\sum_{k=0}^{\infty} a_k$ converges, then $\lim_{k \to \infty} a_k = 0$. Is the converse true? If your answer is "No" provide a counterexample. (10 pts)

3. True or False: If $\sum_{k=0}^{\infty} | a_k |$ converges, then $\sum_{k=0}^{\infty} a_k$ converges. Is the converse true? If your answer is "No" provide a counterexample. (10 pts)

4. Show that the power series $\sum_{k=1}^{\infty} \frac{1}{k^2} x^k$ has interval of convergence [-1,1]. (10 pts)

5. Find the Taylor series of $f(x) = e^{2x}$ at $x_0 = 1$. Make sure you include the $n^{th}$ - term of the series. (10 pts)

6. Find the limits: (a) $\lim_{(x,y)\to(\frac{\pi}{2},0)} \frac{\cos y - 2}{y - \sin x}$ (b) $\lim_{(x,y)\to(2,1)} \frac{xy - 3y - 2x + 1}{x - 1}$ (20 pts)

7. By considering different paths of approach, show that the function $f(x,y) = \frac{x^2 + y}{y}$ has no limit as $(x,y) \to (0,0)$. (10 pts)

8. Check whether the function $f(x,y) = \frac{1}{x^2 - y}$ is continuous at (1,-1). Can you describe all points of discontinuity of $f$ ? (10 pts)

9. Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of the function $f(x,y) = e^{-x}\sin(x+y)$. (10 pts)