

Test time = 8:30 11:30

MAT 200 Fall 2006, Exam 1, Sept 26,

Name

1. The speed of a roller coaster is measured at 1-minute intervals and is given in the table below. Estimate the distance traveled (in feet) during the 3 minutes that data was recorded.

$t$ (minutes)	0	1	2	3
$v(t)$ (feet/minute)	50	47	46	40

$$v \cdot t = d \quad \Delta t = 1$$

$$\text{distance} \approx \sum_{i=1}^3 v(t_i) \cdot \Delta t = 50 + 47 + 46 = 143$$

$$\text{or } 47 + 46 + 40 = 133$$

2. Use the form of the definition of the integral to evaluate the integral

$$\int_{-1}^2 (2-3x) dx$$

$$\Delta x = \frac{2 - (-1)}{n} = \frac{3}{n}$$

$$x_i = -1 + i \Delta x = -1 + i \frac{3}{n} = \frac{3i-n}{n}$$

$$f(x_i) = 2 - 3x_i = 2 - \frac{9i-3n}{n} = \frac{5n-9i}{n}$$

$$f(x_i) \cdot \Delta x = \frac{5n-9i}{n} \cdot \frac{3}{n} = \frac{15n-27i}{n^2}$$

$$\int_{-1}^2 (2-3x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{15n-27i}{n^2}$$

$$= \lim_{n \rightarrow \infty} \left( \sum_{i=1}^n \frac{15n}{n^2} - \sum_{i=1}^n \frac{27i}{n^2} \right)$$

$$= \lim_{n \rightarrow \infty} \left( \frac{15}{n} \sum_{i=1}^n 1 - \frac{27}{n^2} \sum_{i=1}^n i \right)$$

$$= \lim_{n \rightarrow \infty} \left( 15 - \frac{27}{n^2} \cdot \frac{n(n+1)}{2} \right)$$

$$= 15 - \frac{27}{2} = \frac{3}{2}$$

$$\text{check } \int_{-1}^2 (2-3x) dx$$

$$= 2x - \frac{3}{2}x^2 \Big|_{-1}^2 = (4-6) - (-2)$$

~~6~~ (9)

3. Evaluate the integrals.

(5) a.  $\int \frac{x+1}{x} dx = \int 1 + \frac{1}{x} dx = x + \ln|x| + C$

(75) b.  $\int \frac{x+2}{\sqrt{x^2+4x}} dx = \int u^{-1/2} \cdot \frac{1}{2} du = \frac{1}{2} \cdot 2u^{1/2} + C$   
 $u = x^2 + 4x$   
 $du = 2x + 4 dx$   
 $\frac{1}{2} du = (x+2) dx$   
 $= \sqrt{x^2+4x} + C$

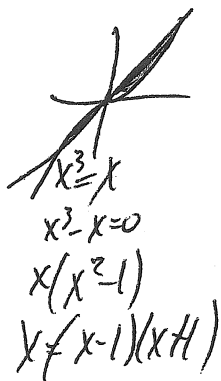
(75) c.  $\int x e^{x/3} dx$   
 $u = x$   
 $du = dx$   
 $v = 3e^{x/3}$   
 $dv = e^{x/3} dx$   
 $\int x e^{x/3} dx = 3x e^{x/3} - \int 3e^{x/3} dx$   
 $= 3x e^{x/3} - 9e^{x/3} + C$

(75) d.  $\int \frac{\ln x}{x^2} dx$   
 $u = \ln x$   
 $du = \frac{1}{x} dx$   
 $v = -\frac{1}{x}$   
 $dv = \frac{1}{x^2} dx$   
 $= -\frac{1}{x} \ln x + \int \frac{1}{x^2} dx$   
 ~~$= -\frac{1}{x} \ln x - \frac{3}{x^3} + C$~~   
 $= -\frac{1}{x} \ln x - \frac{1}{x} + C$

(9)

4.

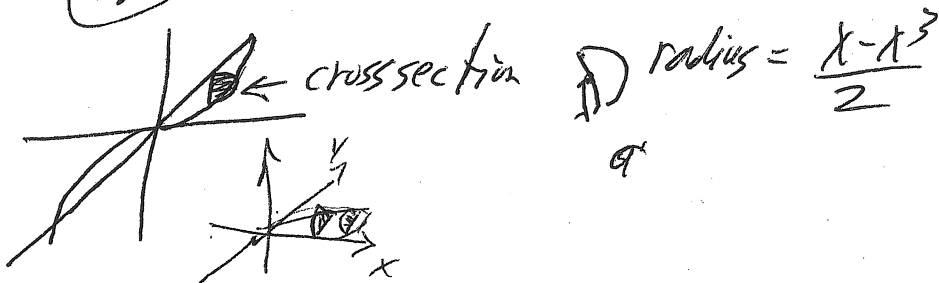
a. Find the area enclosed by the curves  $y = x^3$  and  $y = x$ .



$$\begin{aligned}
 \text{area} &= 2 \int_0^1 (x - x^3) dx \\
 &= 2 \left( \frac{1}{2}x^2 - \frac{1}{4}x^4 \right) \Big|_0^1 \\
 &= 2 \cdot \frac{1}{4} = \frac{1}{2}
 \end{aligned}$$

b. The base of a sculpture is the area enclosed by the curves  $y = x^3$  and  $y = x$ . Cross-sections perpendicular to the  $x$ -axis are semicircles.

i. Draw a diagram of the sculpture. Show the base and a typical cross-section. Label the elements of your diagram appropriately.



ii. Find the volume of the solid  $S$ .

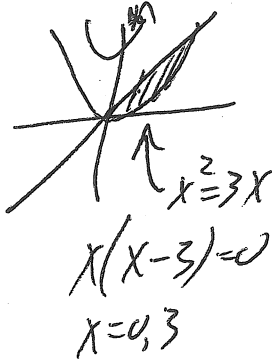
$$\begin{aligned}
 \text{area of cross section} &= \frac{1}{2} \pi r^2 \\
 &= \frac{1}{2} \pi \left( \frac{x - x^3}{2} \right)^2 \\
 &= \frac{\pi}{8} (x^2 - 2x^4 + x^6)
 \end{aligned}$$

$$\begin{aligned}
 \text{Volume} &= 2 \int_0^1 \frac{\pi}{8} (x^2 - 2x^4 + x^6) dx \\
 &= \frac{\pi}{4} \left( \frac{x^3}{3} - \frac{2}{5}x^5 + \frac{1}{7}x^7 \right) \Big|_0^1 \\
 &= \frac{\pi}{4} \left( \frac{1}{3} - \frac{2}{5} + \frac{1}{7} \right) = \frac{\pi}{4} \left( \frac{35 - 42 + 15}{3 \cdot 5 \cdot 7} \right) = \frac{2\pi}{105}
 \end{aligned}$$

~~8~~ 9

5. Find the volume of the solid generated by rotating the region bounded by the given curves about the  $y$ -axis.

$y = x^2, y = 3x$



shells volume =  $\int_a^b 2\pi r h dx$

$$= \int_0^3 2\pi x (3x - x^2) dx$$

$$= 2\pi \int_0^3 (3x^2 - x^3) dx$$

$$= 2\pi \left( x^3 - \frac{1}{4}x^4 \right) \Big|_0^3$$

$$= 2\pi \left( 27 - \frac{3 \cdot 27}{4} \right)$$

$$= 2\pi \cdot \frac{27}{4} = \frac{27}{2} \pi$$

6. On a certain website, the popularity of a recently posted video is modeled by the function  $P(t) = 10e^{-t/24}$ ,  $t$  hours after it is posted. What is the average popularity during the first week it is posted to the website?

~~10 pts~~  
~~5 pts~~  
10 pts

same as 6