Test time = 8:30 11:30

b

MAT 200 Fall 2006, Exam 1, Sept 26, Name______________________

1. The speed of a roller coaster is measured at 1-minute intervals and is given in the table below. Estimate the distance traveled (in feet) during the 3 minutes that data was recorded.

t (minutes)	0	1	2	3
$v(t)$ (feet/minute)	40	46	47	50

midpoints

$43+46.5+48.5 = 138$

$r \cdot t = d \quad \Delta t = 1$

distance $\approx \sum_{i=1}^{3} r(t)\Delta t = 40+46+47 = 133$

or

$= 46+47+50 = 143$

2. Use the form of the definition of the integral to evaluate the integral $\int_{-1}^{2}(3-2x)\,dx$

$\int 3-3x\,dx$

$2x-\frac{3}{2}x^2\Big|_{-1}^{2}$

$(4-6)-(-2-\frac{3}{2})$

$3/2$

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$

$x_i = a + i\Delta x = -1 + i\cdot\frac{3}{n} = \frac{3i-n}{n}$

$f(x_i) = 3-2(x_i) = 3-2\left(\frac{3i-n}{n}\right) = \frac{3n-6i+2n}{n} = \frac{5n-6i}{n}$

$f(x_i)\cdot\Delta x = \frac{5n-6i}{n}\cdot\frac{3}{n} = \frac{15n-18i}{n^2}$

$\int_{-1}^{2} 3-2x\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{15n-18i}{n^2}$

$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{15}{n}-\frac{18i}{n^2}\right) = \lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{15}{n}-\sum_{i=1}^{n}\frac{18i}{n^2}\right)$

$= \lim_{n\to\infty}\left(\frac{15}{n}\sum_{i=1}^{n}1-\frac{18}{n^2}\sum_{i=1}^{n}i\right) = \lim_{n\to\infty}\left(\frac{15}{n}\cdot n-\frac{18}{n^2}\cdot\frac{n(n+1)}{2}\right)$

$= \lim_{n\to\infty}\left(15-\frac{18}{n^2}\cdot\frac{n^2(1+1/n)}{2}\right) = 15-9 = 6$

check: $\int_{-1}^{2} 3-2x\,dx = 3x-x^2\Big|_{-1}^{2} = (6-4)-(-3-1) = 6$

b

3. Evaluate the integrals.

a. $\int \frac{x+2}{x} dx$ $= \int \frac{x}{x} + \frac{2}{x} dx = x + 2\ln|x| + C$

b. $\int \frac{6x^2+8}{\sqrt{x^3+4x}} dx$ $= \int \frac{1}{\sqrt{u}} \cdot 2du = 4u^{1/2} + C$

$= 4\sqrt{x^3+4x} + C$

$u = x^3+4x$

$du = (3x^2+4)dx$

$2du = (6x^2+8)dx$

check

$4 \cdot \frac{1}{2} (x^3+4x)^{-1/2} \cdot 3x^2+4$

c. $\int xe^{x/2} dx$

$u = x$ $\quad v = 2e^{x/2}$

$du = dx$ $\quad dv = e^{x/2}dx$

$\int xe^{x/2}dx = 2xe^{x/2} - \int 2e^{x/2} dx$

$= 2xe^{x/2} - 4e^{x/2} + C$

d. $\int \frac{\ln x}{x^3} dx$ $= -\frac{1}{2} x^{-2} \ln x + \frac{1}{2} \int x^{-3} dx = -\frac{1}{2} x^{-2} \ln x - \frac{1}{4} x^{-2} + C$

$= -\frac{1}{2} x^{-2} \ln x - \frac{1}{4} x^{-2} + C$

~~$u = x^{-3}$ $du = -3x^{-4}dx$ $dv = dx$~~

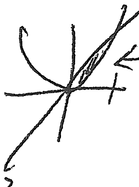
$u = \ln x$ $\quad V = \frac{-1}{2} x^{-2}$

$du = \frac{1}{x}dx$ $\quad dv = x^{-3}dx$

b

4.

a. Find the area enclosed by the curves $y = x^2$ and $y = x$.



$$\text{area} = \int_0^1 x - x^2\,dx$$

$$= \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 \Big|_0^1$$

$$= \tfrac{1}{2} - \tfrac{1}{3} = \tfrac{1}{6}$$

$x^2 = x$

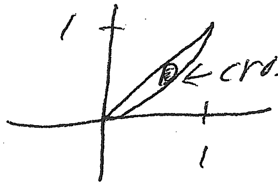
$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

b. The base of a sculpture is the area enclosed by the curves $y = x^2$ and $y = x$. Cross-sections perpendicular to the x-axis are semicircles.

i. Draw a diagram of the sculpture. Show the base and a typical cross-section. Label the elements of your diagram appropriately.



cross section is a semi circle

radius $= \dfrac{x - x^2}{2}$

ii. Find the volume of the solid S.

$$\text{volume} = \int_0^1 \frac{\pi}{2}\left(\frac{x-x^2}{2}\right)^2 dx = \frac{\pi}{8}\int_0^1 x^2 - 2x^3 + x^4\,dx$$

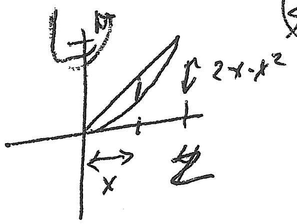
$$= \frac{\pi}{8}\left(\tfrac{1}{3}x^3 - \tfrac{1}{2}x^4 + \tfrac{1}{5}x^5\right)\Big|_0^1$$

$$= \frac{\pi}{8}\left(\tfrac{1}{3} - \tfrac{1}{2} + \tfrac{1}{5}\right)$$

$$= \frac{\pi}{\cancel{120}\ 240}$$

(b)

5. Find the volume of the solid generated by rotating the region bounded by the given curves about the y-axis.
$y = x^2, y = 2x$



shells

$$\text{volume} = \int_0^2 2\pi x(2x - x^2)\,dx$$
$$= 2\pi \int_0^2 2x^2 - x^3\,dx$$
$$= 2\pi\left(\frac{2}{3}x^3 - \frac{1}{4}x^4\right)\Big|_0^2$$
$$= 2\pi\left(\frac{2}{3}\cdot 2^3 - 2^2\right) = \frac{8\pi}{3}$$

$x^2 = 2x$
$x^2 - 2x = 0$
$x(x-2) = 0$

disks

$$\text{volume} = \int_0^4 \pi\left((\sqrt{y})^2 - \left(\frac{y}{2}\right)^2\right)dy$$

$x = \sqrt{y}$
$x = \frac{y}{2}$

$$= \pi\int_0^4 y - \frac{y^2}{4}\,dy$$
$$= \pi\left(\frac{1}{2}y^2 - \frac{1}{12}y^3\right)\Big|_0^4 = \pi\left(8 - \frac{16}{3}\right) = \frac{8\pi}{3}$$

6. On a certain website, the popularity of a recently posted video is modeled by the function $P(t) = 10e^{-t/24}$, t hours after it is posted. What is the average popularity during the first week it is posted to the website?

1 week = 7 days = $7 \cdot 24$ hours

$$\text{avg pop} = \frac{1}{7\cdot 24}\int_0^{7\cdot 24} 10e^{-t/24}\,dt$$
$$= \frac{1}{7\cdot 24}\left(-240\,e^{-t/24}\right)\Big|_0^{7\cdot 24}$$
$$= \frac{1}{7\cdot 24}\left(-240\cdot e^{-7} + 240\right)$$
$$= \frac{10}{7} - \frac{10}{7}e^{-7} = \frac{10}{7}\left(1 - e^{-7}\right)$$