1. Let $P(x)$ be the statement "$x^2 \geq |x|$", and the universe of discourse be real numbers. What are the truth values of the following? [Explain.]

   (a) $P(2)$ (b) $\exists x\, P(x)$ (c) $\forall x\, P(x)$

2. If $Q(x, y)$ is the statement "$x - y = x + y$" and the universe of discourse is the set of real numbers, determine the truth value of the following statements.
   [Explain.]

   (a) $\forall x \exists y\; Q(x, y)$ (b) $\exists x \forall y\; Q(x, y)$

   (c) $\forall y \exists x\; Q(x, y)$ (d) $\exists y \forall x\; Q(x, y)$

3. Determine whether the following arguments are valid, giving counterexamples where necessary:

   (a) All tall people sit in the back of class. Jim sits in the back hence Jim is tall.

   (b) If a and b are rational numbers then $a+b$ is rational. Suppose a and b are not rational then $a+b$ is not rational.

4. Complete the following argument (and explain your reasoning):

$$\begin{array}{c} p \rightarrow q \\ r \rightarrow \neg s \\ \neg(q \vee \neg s) \\ \hline \therefore \boxed{} \end{array}$$

5. Show that $\neg\left(p \vee (\neg p \wedge q)\right)$ and $\neg p \wedge \neg q$ are logically equivalent

(a) using a truth table

p	q	
T	T	
T	F	
F	T	
F	F	

(b) and also without using a truth table

$\neg\left(p \vee (\neg p \wedge q)\right) \equiv$ ....

....

....

6. (a) What is the negation of $\forall x \exists y : x + y$ is odd .

(b) What is the contra-positive of If you are rich you have many friends.

7. (a) Show that $\left[(p \vee q) \wedge (\neg p \vee r)\right] \rightarrow (q \vee r)$ is a tautology.

(b) What does this imply for the argument

$$\begin{array}{c} p \vee q \\ \neg p \vee r \\ \hline \therefore \ \boxed{} \end{array}$$

$\therefore$

8. What conclusion(s) can be drawn from the following set of premises:

   "Every computer science major has a computer or takes math classes."
   "Bill doesn't have a computer and doesn't take math classes"
   "Susan doesn't have a computer but does take math classes"
   "All Bill's friends are computer science majors who don't take math classes"

9. Use rules of inference to show that if $\forall x\left[P(x) \rightarrow \left(R(x) \wedge S(x)\right)\right]$ and $\forall x\left[P(x) \wedge T(x)\right]$ are true then $\forall x\left[T(x) \wedge S(x)\right]$ is true [Use as many lines as you need]

   1. $\forall x\left[P(x) \wedge T(x)\right]$ Premise
   2. ....
   3. ....
   4. $P(a)$
   5. $\forall x\left[P(x) \rightarrow \left(R(x) \wedge S(x)\right)\right]$ Premise
   6. ....
   7. ....
   8. ....
   9. ....
   10. ....

10. Prove, or disprove, that the product of two irrational numbers is irrational.
(*Note*: If you are disproving, provide a counterexample)

11. Let n be an integer. Prove that if $3n^2 + 7$ is odd, then n is even.
(*Hint*: Use proof by contradiction)

12. Let n be a positive integer. Prove or disprove that the equation $n^3 + n^2 = 150$ has at most one solution.