TEST 2

(Math 258)

1. (i) Let $A = \{1, 2, 5, 7\}$ and $B = \{2, 3, 7, 9, 10\}$. Find: (40 pts)

   (a) $A \cup B$ (b) $A \cap B$

   (c) $A - B$ (d) $B - A$

   (ii) Let $A = \{\varnothing\}$ and $B = \{\varnothing, \{\varnothing\}\}$. Find:

   (a) $P(A)$ (b) $P(P(A))$

   (c) $P(\varnothing)$ (d) $B \times B$

2. Determine which of the following sets is the power set of a set. For the ones that are power sets, write down the set. (10 pts)

   (a) $\varnothing$ (b) $\{\varnothing, \{a\}\}$

3. (a) Show that if $A \subseteq B$, then $A \cup B = B$. (10 pts)

   (b) Find two sets A and B such that $A \in B$ and $A \subseteq B$.

4. (a) Show that if $A \subseteq B$, then $P(A) \subseteq P(B)$. (10 pts)

   (b) Show that for any sets A and B, we have $A - B = A \cap \overline{B}$.

5. Let A and B be subsets of a universal set U. Show that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$. (10 pts)
   (*Hint*: Establish the above using a Proof By Contradiction)

6. The **symmetric difference** of two sets A and B, denoted by $A \oplus B$, is the set containing those elements that belong to either A or B, but not in both. Answer the following: (10 pts)

   (a) Show that $A \oplus U = \overline{A}$ (b) If $A \oplus B = A$, what can you say about B? Explain.

7. The **successor** of a set A is the set $A \cup \{A\}$. Find the successors of the following sets: (10 pts)

   (a) $\varnothing$ (b) $\{\varnothing\}$ (c) $\{\varnothing, \{\varnothing\}\}$

   How many elements does the successor of a set with n elements have?