MAT 140 **Problems Week 1 Solutions** **Fall 2007**

1. Given the function $f(x) = \frac{x^2 - 1}{x+1}$, find the following:

(a) Find the value of $f(2)$ (b) Find the domain of f (*Note*: Do <u>not</u> do the graph)

(c) Is the point (4, 7) on the graph of f ?

Solution: (a) $f(2) = \frac{2^2 - 1}{2+1} = \frac{4-1}{2+1} = \frac{3}{3} = \boxed{1}$

(b) Since we need $x+1 \neq 0$ we find $\boxed{D_f = \mathbb{R} \setminus \{-1\}}$

(c) For the point (4, 7) to be on the graph of f we would need $f(4) = 7$,

but $f(4) = \frac{4^2 - 1}{4+1} = \frac{16-1}{4+1} = \frac{15}{5} = 3$. Hence: $\boxed{\text{No}}$

2. Find the domains of (a) $f(x) = \sqrt{9 - x^2}$ (c) $h(x) = \frac{\sqrt{9 - x^2}}{\ln(x)}$

(b) $g(x) = \ln(x)$

Solution: (a) $9 - x^2 \geq 0 \Rightarrow -3 \leq x \leq 3$. Hence: $\boxed{D_f = [-3, 3]}$

(b) $\boxed{D_g = (0, \infty)}$

(c) Combining the domains from (a) and (b), *and* realizing that also the denominator cannot be zero [i.e. $\ln(x) \neq 0 \Rightarrow x \neq 1$] we find

$\boxed{D_h = (0, 1) \cup (1, 3]}$

3. Let $f(x) = \frac{x}{x+1}$ and $g(x) = \sqrt{x^2 - 4}$. Find

(a) $f + g$ (b) $f \div g$ (c) $f \circ g$ (d) $g \circ f$ (e) $g \circ g$

Solution: (a) $(f+g)(x) = \boxed{\frac{x}{x+1} + \sqrt{x^2 - 4}}$

(b) $\frac{f}{g}(x) = \frac{\frac{x}{x+1}}{\sqrt{x^2-4}} = \boxed{\frac{x}{(x+1)\sqrt{x^2-4}}}$

(c) $(f \circ g)(x) = f\left(g(x)\right) = \boxed{\frac{\sqrt{x^2-4}}{\sqrt{x^2-4}+1}}$

(d) $(g \circ f)(x) = g\left(f(x)\right) = \boxed{\sqrt{\left(\frac{x}{x+1}\right)^2 - 4}}$

(e) $(g \circ g)(x) = g\left(g(x)\right) = \sqrt{\left(\sqrt{x^2-4}\right)^2 - 4} = \boxed{\sqrt{x^2-8}}$

7. Find the inverse, f^{-1}, of $f(x) = \sqrt[3]{\frac{2x+1}{5}}$.

Solution: For f^{-1} we have $x = \sqrt[3]{\frac{2y+1}{5}} \Rightarrow x^3 = \frac{2y+1}{5}$

$\Rightarrow 5x^3 = 2y+1$

$\Rightarrow 5x^3 - 1 = 2y$

Hence $\boxed{f^{-1}(x) = \frac{5x^3-1}{2}}$

8. Prove the following identities:

(b) $\tan(x)\sin(2x) = 1 - \cos(2x)$ (c) $\frac{2\sin(x)}{\cos(x)+\sin(x)} = \tan(2x) + 1 - \sec(2x)$

Solutions:

(a) LHS $= \tan(x)\sin(2x) = \frac{\sin(x)}{\cos(x)} 2\sin(x)\cos(x) = 2\sin^2(x) = 1 - \cos(2x) =$ RHS

(b) $$\text{LHS} = \frac{2\sin(x)}{\cos(x)+\sin(x)} = \frac{2\sin(x)}{\cos(x)+\sin(x)}\cdot\frac{\cos(x)-\sin(x)}{\cos(x)-\sin(x)}$$
$$= \frac{2\sin(x)\big(\cos(x)-\sin(x)\big)}{\cos^2(x)-\sin^2(x)} = \frac{2\sin(x)\cos(x)-2\sin^2(x)}{\cos(2x)}$$
$$= \frac{\sin(2x)-1+1-2\sin^2(x)}{\cos(2x)} = \frac{\sin(2x)-1+\cos(2x)}{\cos(2x)}$$
$$= \tan(2x)+1-\sec(2x) = \text{RHS}$$

9. Solve: (a) $2\cos(x)-1=0$ for $0 \le x \le 2\pi$

Solutions: **(a)** $$2\cos(x)-1=0 \;\Rightarrow\; \left.\begin{array}{r}\cos(x)=\frac{1}{2}\\ 0\le x\le 2\pi\end{array}\right\} \;\Rightarrow\; \boxed{x=\frac{\pi}{3} \text{ or } \frac{5\pi}{3}}$$

12. Let $\cos(A) = -\frac{12}{13}$, where A is an angle in quadrant III, and $\cos(B)=\frac{3}{5}$, where B be an angle in quadrant IV. Compute:

(a) $\cos(A+B)$ (b) $\cos(2B)$ (c) $\tan(2B)$

Solutions: $$\cos(A) = -\frac{12}{13} \text{ with } A\in\text{III} \;\Rightarrow\; \sin(A) = -\sqrt{1-\left(-\tfrac{12}{13}\right)^2} = -\frac{5}{13}$$
$$\cos(B)=\frac{3}{5} \text{ with } B\in\text{IV} \;\Rightarrow\; \sin(B) = -\sqrt{1-\left(\tfrac{3}{5}\right)^2} = -\frac{4}{5}$$

Hence:

(a) $$\cos(A+B) = \cos(A)\cos(B)-\sin(A)\sin(B) = \left(-\tfrac{12}{13}\right)\left(\tfrac{3}{5}\right)-\left(-\tfrac{5}{13}\right)\left(-\tfrac{4}{5}\right) = \boxed{-\tfrac{56}{65}}$$

(b) $$\cos(2B) = \cos^2(B)-\sin^2(B) = \left(\tfrac{3}{5}\right)^2-\left(-\tfrac{4}{5}\right)^2 = \boxed{-\tfrac{7}{25}}$$

(c) $$\tan(2B) = \frac{\sin(2B)}{\cos(2B)} = \frac{2\sin(B)\cos(B)}{\cos(2B)} = \frac{2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right)}{-\frac{7}{25}} = \boxed{\tfrac{24}{7}}$$