TEST 1

(Math 140)

Solve the following problems. Show all your work in the space under each problem.

1. (a) Find the distance between points $P = (2, -1)$ and $Q = (1, 3)$. (15 pts)

(b) Find the midpoint of the line segment joining the points P and Q.

(c) Find the equation of the line passing through the points P and Q .

2. Let $\vec{u} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Find: (10 pts)

(a) $2(\vec{u} + \vec{w}) - 3\vec{v}$

(b) $\|\vec{w}\|$

3. The vector $\vec{u}$ has initial point $P = (2, -3)$ and terminal point $Q = (-1, 2)$. (15 pts)

(a) Find its position vector.

(b) Find the unit vector having the same direction as $\vec{u}$ (i.e. the "direction vector").

(c) Write $\vec{u}$ in the form $\vec{u} = a\vec{\varepsilon}_1 + b\vec{\varepsilon}_2$, where $\vec{\varepsilon}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{\varepsilon}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ respectively.

4. (a) Find the angle between the vectors $\vec{u} = \begin{pmatrix} 2 \\ \sqrt{12} \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} \sqrt{2}/4 \\ 0 \end{pmatrix}$.

(b) Find the orthogonal projection of $\vec{v}$ onto $\vec{u}$. (10 pts)

5. (a) Show that the vectors $\vec{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ are linearly dependent.

(30 pts)

(b) Show that the vectors $\vec{u}$ and $\vec{v}$ above are linearly independent.

(c) Show that $\mathbb{R}^2$ is spanned by the vectors $\vec{u}$ and $\vec{v}$ above.

(d) (i) Do the vectors $\vec{u}$ and $\vec{v}$ above form a basis for $\mathbb{R}^2$?
(ii) Do they form an orthogonal basis ?
(iii) What is the dimension of $\mathbb{R}^2$?

6. (a) Exhibit a basis for the vector space $M_{2\times 2}(\mathbb{R})$, other than the “natural” basis.

(20 pts)

(b) The dimension of $M_{2\times 2}(\mathbb{R})$ is: A. 2 B. 3 C. 4 D. 8 .

(c) **True** or **False**: The following vectors can *__not__* be a basis for $P_2[x]$:

$$p(x) = 1 + x \ , \quad q(x) = 2 + 2x \ , \quad r(x) = x + x^2$$

(d) **True** or **False**: The dimension of $P_2[x]$ is 3.