HOMEWORK 1

(Math 140)

1. What is the dimension of $\mathbb{C}$, the vector space of the complex numbers? Why? (10 pts)
   (*Note*: The scalars are taken to be in **R**, i.e. they are real numbers)

2. Show that the polynomials $p_1(x) = 1 + x$, $p_2(x) = 1 - x$, $p_3(x) = x + x^2$ form a basis for the vector space $P_2[x]$, the space of second-degree polynomials. (10 pts)

3. Given $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, find a formula for A^n, where *n* is a positive integer. (10 pts)

4. Given $A = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$, show that $A^3 = I$. Use that to find A^{-1}. (20 pts)
   (*Note*: Do <u>not</u> use the formula for A^{-1} to find A^{-1})

5. Given that *A* and *B* are $n \times n$ matrices, show that $(A+B)(A-B) = A^2 - B^2$ if and only if $\mathrm{AB} = \mathrm{BA}$. (10 pts)

6. Given the map $A: \mathbf{R}^2 \to \mathbf{R}^2$, defined by $A\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_1 + u_2 \\ -u_2 \end{pmatrix}$, do the following:
   (a) Show that *A* is a linear map (30 pts)
   (b) Find K_A, the kernel of *A*
   (c) Find the matrix that expresses the linear map *A*

   (*Note*: We denote the corresponding matrix in (c), also by *A*)

7. If the area of a parallelogram, generated by $\vec{u}$ and $\vec{v}$, is equal to 2 square units, then what is the area of the new parallelogram formed after we apply matrix $A = \begin{pmatrix} 4 & 1 \\ 3 & 1 \end{pmatrix}$ on the vectors $\vec{u}$ and $\vec{v}$? (10 pts)