HOMEWORK 1

(Math 258 A,B)

1. Let $f(x) = ax + 1$ and $g(x) = 2x + a + 2$, where a is a real number. Find for which a's we have $f \circ g = g \circ f$. (10 pts)

2. Let $f : A \to B$ and $S \subseteq A$, $T \subseteq A$. Show that: (30 pts)
   (a) $f(S \cup T) = f(S) \cup f(T)$
   (b) $f(S \cap T) \subseteq f(S) \cap f(T)$
   (c) Give an example of a function *f* and sets *S* and *T* such that $f(S \cap T) \neq f(S) \cap f(T)$

3. Let $f : A \to B$ and $S \subseteq B$. The **inverse image** of *S* is defined by: (20 pts)

$$f^{-1}(S) = \{a \in A \mid f(a) \in S\}$$

   In other words, is the subset of *A* containing the pre-images of all elements of *S*. Let f be a real function defined by $f(x) = x + 1$. Find:
   (a) $f^{-1}(\{0\})$
   (b) $f^{-1}(\{x \mid 0 < x < 1\})$

4. Given $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, find a formula for A^n, where *n* is a positive integer. (10 pts)

5. Given $A = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$, show that $A^3 = I$. Use that to find A^{-1}. (20 pts)
   (*Note*: Do <u>not</u> use the formula for A^{-1} to find A^{-1})

6. Given that *A* and *B* are $n \times n$ matrices, show that $(A + B)(A - B) = A^2 - B^2$ if and only if $\mathrm{AB} = \mathrm{BA}$. (10 pts)