HOMEWORK 2

(Math 258 A,B)

1. Prove **Pascal's Identity**. Namely, show that: (20 pts)

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}, \text{ where } k \leq n.$$

(*Note*: Prove the identity by algebraic methods, <u>not</u> combinatorial methods).

2. Prove the **Hexagon Identity**. Namely, show that: (20 pts)

$$\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n}{k-1}\binom{n+1}{k+1}, \text{ where } 1 \leq k < n.$$

Examine how you can use the identity above to find terms in Pascal's triangle that form a hexagon.

(*Note*: Prove the identity by algebraic methods, <u>not</u> combinatorial methods).

3. Establish **Bolferoni's Inequality**. Namely, show that: (20 pts)

$$P(E \cap F) \geq P(E) + P(F) - 1, \text{ where } E \text{ and } F \text{ events.}$$

4. Two events E and F are called **independent** if and only if $P(E \cap F) = P(E)P(F)$. Show that if E and F are independent, then $\overline{E}$ and $\overline{F}$ are also independent. (20 pts)

5. Determine whether the relation $R = \{(x, y) \in \Re \times \Re \mid xy \geq 0\}$ is: (20 pts)

(a) reflexive (b) symmetric (c) antisymmetric (d) transitive