# Test 1

## (Math 140)

1. Given the function $f(x) = \dfrac{x+1}{x-3}$, find the following:
   a. Find the value of f(1)
      f(1) = $\dfrac{1+1}{1-3}$ = 2 / -2 = -1
   b. Solve the equation f(x) = 2
      $2 = \dfrac{x+1}{x-3} \quad \Rightarrow x+1 = 2x-6 \Rightarrow x = 7$
   c. Is the point (4,7) on the graph of f?
      $7 = \dfrac{4+1}{4-3} \Rightarrow 7 = 5$. So (4,7) is not on the graph

   d. What is the domain of f?
      $D = \{x \in \Re : x \neq 3\}$

2. Given the function f(x) = 3x and g(x) = 1 – $x^2$ , find the following:

   a. $f \circ g(x)$
      $f \circ g(x) = f(g(x)) = 3(1 - x^2)$

   b. $g \circ f(1)$
      $g \circ f(1) = g(f(1)) = g(3) = 1 - 3^2 = -8$

3. Given the function f(x) = 2x + 1, find the following
   a. Check whether f is one-to-one
      $f(x_1) = f(x_2)$
      $2x_1 + 1 = 2x_2 + 1$
      $2x_1 = 2x_2$
      $x_1 = x_2$
      So f is one-to-one

b. Find $f^{-1}$

$y = 2x + 1$

$x = 2y + 1$

$$y = \frac{x-1}{2}$$

$$f^{-1} = \frac{x-1}{2}$$

4. Do the following problems:

a. If $\cos\theta = \frac{\sqrt{2}}{\sqrt{11}}, \frac{3\pi}{2} < \theta < 2\theta$, find $\csc\theta$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$= 1 - \frac{2}{11} = 9/11$$

$$\sin\theta = -\sqrt{\frac{9}{11}} \Rightarrow \frac{1}{\sin\theta} = \csc\theta = -\sqrt{\frac{11}{3}} = -\frac{\sqrt{11}}{3}$$

b. Find the exact value of $\sin(\tan^{-1}\frac{1}{2})$

$$\tan^{-1}\frac{1}{2} = \theta \Rightarrow \tan\theta = \frac{1}{2}$$

$$\sec^2\theta = 1 + \tan^2\theta = 1 + \frac{1}{4} = \frac{5}{4}$$

$$\sec\theta = \frac{\sqrt{5}}{2} \Rightarrow \cos\theta = \frac{2}{\sqrt{5}}$$

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{2}{\sqrt{5}}\right)^2 = 1 - \frac{4}{5} = \frac{1}{5} \Rightarrow \sin\theta = \frac{1}{\sqrt{5}}$$

5. Establish the identity $\tan\theta + \frac{\cos\theta}{1+\sin\theta} = \sec\theta$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{\sin\theta(1+\sin\theta) + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\sin\theta + 1}{\cos\theta(1+\sin\theta)} = \frac{1}{\cos\theta} = \sec\theta$$

6. Find the exact value of the following:

a. $\sin\frac{\pi}{12}$

$$\sin\frac{\pi}{12}=\sin\frac{\pi}{3}-\frac{\pi}{4}=\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$=\frac{\sqrt{3}}{2}*\frac{\sqrt{2}}{2}-\frac{1}{2}*\frac{\sqrt{2}}{2}=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}=\frac{\sqrt{6}-\sqrt{2}}{4}$$

b. $\cos 22.5^\circ$

$$\cos 22.5=\sqrt{\frac{1+\cos 45}{2}}=\sqrt{\frac{1+\sqrt{2}/2}{2}}=\sqrt{\frac{2+\sqrt{2}}{4}}=\frac{\sqrt{2+\sqrt{2}}}{2}$$

7. Establish the identity $\frac{\sec^2\theta}{2-\sec^2\theta}=\sec 2\theta$

$$\text{LHS}=\frac{\frac{1}{\cos^2\theta}}{2-\frac{1}{\cos^2\theta}}=\frac{\frac{1}{\cos^2\theta}}{\frac{2\cos^2\theta-1}{\cos^2\theta}}=\frac{1}{2\cos^2\theta-1}=\frac{1}{\cos 2\theta}=\sec 2\theta$$

8. Solve the equation $\cos 2\theta=-\cos\theta$ in the interval $0\le\theta\le 2\pi$

$$\cos 2\theta=-\cos\theta\Rightarrow 2\cos^2\theta-1=-\cos\theta\Rightarrow 2\cos^2\theta-1+\cos\theta=0$$

$$\cos\theta=\frac{-1\pm\sqrt{1+8}}{4}\Rightarrow\cos\theta=\frac{-1+3}{4}=\frac{1}{2}\text{ and }\cos\theta=\frac{-1-3}{4}=-1$$

$$\cos\theta=\frac{1}{2}\Rightarrow\cos\theta=\cos\frac{\pi}{3}\Rightarrow\theta=2kn\pm\frac{\pi}{3}\Rightarrow\theta=\frac{\pi}{3},\frac{5\pi}{6}$$

$$\cos\theta=-1\Rightarrow\cos\theta=\cos\pi\Rightarrow\theta=2kn\pm\pi\Rightarrow\theta=\pi$$

So, $\theta=\frac{\pi}{3},\frac{5\pi}{6},\pi$