Test 2

(Math 258 A)

1. (i) Let A = { a, b, c, d, e, f} and $B = \{p, d, k, f\}$. Find:

   a. $A \cup B = ?$
   $A \cup B = \{a, b, c, d, e, f, k, p\}$

   b. $A \cap B = ?$
   $A \cap B = \{d, f\}$

   c. $A - B = ?$
   $A - B = \{a, b, c, e\}$

   d. $B - A = ?$
   $B - A = \{p, k\}$

   (ii) Let $A = \{1, 2, 3\}$ and $B = \{\text{a}, \text{b}\}$. Find

   a. $P(A) = ?$
   $P(A) = \{0, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$

   b. $A \times B = ?$
   $A \times B = \{ \{1, a\}, \{1, b\},$
   $\{2, a\}, \{2, b\},$
   $\{3, a\}, \{3, b\} \}$

   (iii) Let $A = \{\varnothing, \{\varnothing\}\}$. Find:

   a. $P(A) = ?$
   $P(A) = \{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$

   b. $P(P(A)) = ?$
   $P(P(A))$ = the set that consists of the following elements:

1	$\varnothing$
2	$\{\varnothing\}$
3	$\{\{\varnothing\}\}$
4	$\{\{\{\varnothing\}\}\}$
5	$\{\{\varnothing, \{\varnothing\}\}\}$
6	$\{\varnothing, \{\varnothing\}\}$
7	$\{\varnothing, \{\{\varnothing\}\}\}$
8	$\{\varnothing, \{\varnothing, \{\varnothing\}\}\}$

9	$\{\{\varnothing\}, \{\{\varnothing\}\}\}$
10	$\{\{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$
11	$\{\{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$
12	$\{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}\}$
13	$\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$
14	$\{\{\varnothing\}, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$
15	$\{\varnothing, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$
16	$\{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$

2. Determine which of the following sets is the power set of a set. For the ones that are power sets, write down the set.

   a. $\{\varnothing, \{a\}\}$
   This is a power set. The set is $\{a\}$.
   b. $\{\varnothing, \{a\}, \{\varnothing, a\}$
   This isn't a power set. It's missing $\{\varnothing\}$.

3. Show that if $A \subseteq B$, then $A \cap B = A$.

   Let $x \in A \cap B \Rightarrow x \in A \wedge x \in B$
   $\Rightarrow x \in A$
   i.e. $A \cap B \subseteq A$.

   Let $x \in A \Rightarrow x \in B$, because $A \subseteq B$
   So, since $x \in A$ and $x \in B \Rightarrow x \in A \cap B$

   i.e. $A \subseteq A \cap B$.

   Since $A \cap B \subseteq A$ and $A \subseteq A \cap B$, we have that $A \cap B = A$.

4. Show that if $A \subseteq B$, then $P(A) \subseteq P(B)$.

   Let $S \in P(A) \Rightarrow S \subseteq A$
   $\Rightarrow S \subseteq B$, since $A \subseteq B$
   $\Rightarrow S \in P(B)$

   i.e. $P(A) \subseteq P(B)$.

5. Show that for any sets A and B, we have $A - B = A \cap \overline{B}$.

$$
\begin{aligned}
A - B &= \{x \mid x \in A - B\} \\
&= \{x \mid x \in A \wedge x \notin B)\} \\
&= \{x \mid x \in A \wedge x \in \overline{B}\} \\
&= \{x \mid x \in A \cap \overline{B}\}
\end{aligned}
$$

6. The **symmetric difference** of the two sets A and B, denoted by $A \oplus B$, is the set containing those elements that belong to either A or B, but not in both. Answer the following:

   a. Show that $\text{A} \oplus \varnothing = A$

$$
\begin{aligned}
\text{A} \oplus \varnothing &= \{x \mid (x \in A \wedge x \notin \varnothing) \vee (x \notin A \wedge x \in \varnothing)\} \\
&= \{x \mid (x \in A \wedge \mathbf{T}) \vee (x \notin A \wedge \mathbf{F})\} \\
&= \{x \mid x \in A \vee \mathbf{F}\} \\
&= \{x \mid (x \in A)\} \\
&= A
\end{aligned}
$$

   b. If $A \oplus B = A$, what can you say about B? Explain.

   B must be the empty set. If *B* had anything other than the empty set in it, say *x*, this *x* could either be in *A* or not.
   Case 1: If $x \in A$, then it wouldn't be included in the final outcome since it is defined as the set of all elements that belong to either *A* or *B*, not both, and it the final outcome wouldn't equal *A*, so contradiction.

   Case 2: If *x* isn't in *A*, then it would be included in the final outcome and it wouldn't equal only *A*, so another contradiction.