TEST 2

(Math 258 B)

1. (i) Let $A = \{s, p, q, r\}$ and $B = \{a, p, b, c, r, e\}$. Find (40 pts)

(a) $A \cup B$
$\{s, p, q, r, a, b, c, e\}$

(b) $A \cap B$
$\{p, r\}$

(c) $A - B$
$\{s, q\}$

(d) $B - A$
$\{a, b, c, e\}$

(ii) Let $A = \{a, b, c\}$ and $B = \{1, 2\}$. Find

(a) P(A) = {Ø, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

(b) $A \times B = \{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$

(iii) Let $A = \{Ø, \{Ø\}\}$

(a) P(A) = {Ø, {Ø, {Ø}}, {Ø}, {{Ø}}}

(b) P(P(A)) = P({Ø, {Ø, {Ø}}, {Ø}, {{Ø}}}) =

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}

Note: the power set with n elements will have 2^n elements, n = 4, so there are 16 elements

2. Determine which of the following sets is the power set of a set. (10 pts)
For the ones that are power sets, write down the set.

(a) $\{\{\varnothing\},\{a\},\{\varnothing,a\}\}$
not a power set

(b) $\{\varnothing,\{\{a\}\}\}$
Yes, A = $\{\{a\}\}$

3. Show that if $A \subseteq B$ then $A \cup B = \mathrm{B}$ (10 pts)

let $x \in A \cup B$
$\Leftrightarrow x \in A \wedge x \in B$
$\Leftrightarrow x \in B$ (Since $A \subseteq B$)
Hence, any arbitrary $x \in A \cup B$
$\therefore$ if $A \subseteq B$ then $A \cup B = \mathrm{B}$

4. Show that if A $\subseteq B$, then P(A) $\subseteq$ P(B) (10 pts)

Let $S \in P(A)$
$\Rightarrow S \subseteq A$
$\Rightarrow S \subseteq B$ (because A $\subseteq B$)
$\Rightarrow S \in P(B)$ (because $x \in B$)

$\therefore$ P(A) $\subseteq$ P(B)

5. Show that for any sets A and B, we have, $(A - B) = (A \cap \overline{B})$ (10 pts)

Let $x \in (A - B)$
$\Leftrightarrow x \in A \wedge x \notin B$
$\Leftrightarrow x \in A \wedge x \in \overline{B}$
$\Leftrightarrow x \in (A \cap \overline{B})$**,** proving that $(A - B) = (A \cap \overline{B})$

6. (a) Show that $(A \oplus \varnothing) = A$ (10 pts)

Let $x \in (A \oplus \varnothing)$, then
$\Leftrightarrow (x \in A) \wedge (\mathrm{x} \notin \varnothing)$
(because $(A \oplus \varnothing) = (x \in A \vee x \in \varnothing) \wedge (\mathrm{x} \notin \mathrm{A} \cap \varnothing)$), and $\varnothing$ has no elements)
$\Leftrightarrow (x \in A) \wedge \mathrm{T}$ (because $\varnothing$ has no elements)

$\Leftrightarrow (x \in A)$

Therefore $(A \oplus \varnothing) = A$

(b) If $(A \oplus B) = A$, what can you say about B (10 pts)

B must be $\varnothing$

Assume, $\mathrm{B} \neq \varnothing \Rightarrow \exists\, \mathrm{y} \in \mathrm{B}$

Case 1: If $\mathrm{y} \in \mathrm{A} \Rightarrow \mathrm{y} \notin \mathrm{A} \oplus \mathrm{B}$ (because y would be in $A \cap B$)

$\Rightarrow \mathrm{y} \notin \mathrm{A}$ (because $(A \oplus B) = A$)

$\Rightarrow$ contradiction

Case 1: If $\mathrm{y} \notin \mathrm{A} \Rightarrow \mathrm{y} \in \mathrm{A} \oplus \mathrm{B}$ (because $\mathrm{y} \in \mathrm{B}$)

$\Rightarrow \mathrm{y} \in \mathrm{A}$ (because $(A \oplus B) = A$)

$\Rightarrow$ contradiction

All cases result in a contradiction, so $(A \oplus B) = A$