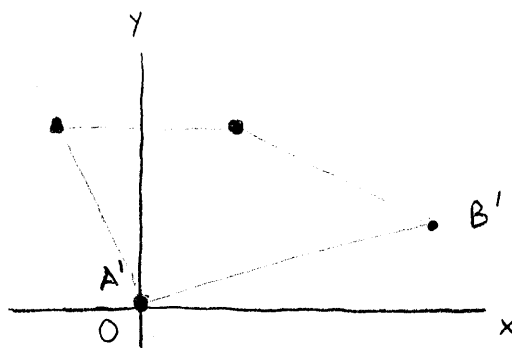
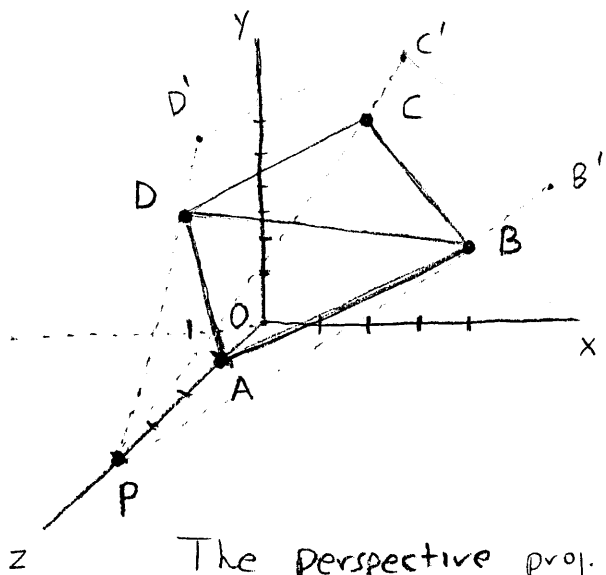


Bonus Quiz

(Math 140)

Problem: Let S be the reversed pyramid with vertices $A = (0,0,1)$, $B = (4,2,2)$, $C = (2,6,0)$, and $D = (-1,3,2)$. A small lamp which is placed in front of the pyramid at $P = (0,0,4)$ casts a shadow on the wall behind the pyramid. Find the coordinates that determine the shadow of the pyramid. Graph the pyramid and its shadow on two different coordinate systems. Find the perimeter of the shadow. (3 pts)



The perspective proj. matrix M is given by : $M = \begin{pmatrix} \frac{1}{1-z/4} & 0 & 0 & 0 \\ 0 & \frac{1}{1-z/4} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

For A' : $\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{1-1/4} & 0 & 0 & 0 \\ 0 & \frac{1}{1-1/4} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4/3 & 0 & 0 & 0 \\ 0 & 4/3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

ie, $A' = (0, 0, 0)$

For B' : $\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{1-2/4} & 0 & 0 & 0 \\ 0 & \frac{1}{1-2/4} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \\ 0 \\ 1 \end{pmatrix}$

ie, $B' = (8, 4, 0)$

For C' :

ie, $C' = (2, 6, 0)$

For D' :

ie, $D' = (-2, 6, 0)$

Perimeter of $A'B'C'D' = A'B' + B'C' + C'D' + D'A'$
 $= \dots$

where, say $A'B'$, equals $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$A' = (x_1, y_1)$
 $B' = (x_2, y_2)$