

TEST 1

(Math 258, A)

- Let $P(x)$ be the statement “ $x = x^2$.” If the universe of discourse consists of the integers, what is the truth value of the following? Explain. (15 pts)
(a) $P(-1)$ (b) $\exists xP(x)$ (c) $\forall xP(x)$
- Determine the truth value of the following statements, if the universe of discourse consists of all real numbers. Explain. (15 pts)
(a) $\forall x\exists y(x = y^2)$ (b) $\forall x\exists y(xy = 0)$ (c) $\exists x\exists y[(x + 2y = 2) \wedge (2x + 4y = 5)]$
- Determine whether the following arguments are valid, giving counterexamples where necessary: (10 pts)
(a) If x is a real number such that $x > 5$, then $x^2 > 25$.
Suppose that $x \leq 5$, then $x^2 \leq 25$.
(b) If x^2 is irrational, then x is irrational.
Suppose x is irrational, then x^2 is irrational.
- Use a truth table to show that the following argument is valid: (15 pts)

$$\begin{array}{l} p \rightarrow q \\ p \rightarrow r \\ \neg(p \wedge q) \\ \hline \neg p \end{array}$$

5. Prove, or disprove, that the product of two irrational numbers is irrational. (10 pts)
(*Note:* If you are disproving, provide a counterexample)

6. Prove that if $n^3 + 5$ is odd, where n is an integer, then n is even. (10 pts)
(*Hint:* Use proof by contradiction)

7. Prove that if a , b and c are real numbers, then $\min(a, \min(b, c)) = \min(c, \min(a, b))$.
(*Hint:* Use proof by cases) (10 pts)

8. Show that, for integers, the following statements are equivalent: (15 pts)
(*Hint:* consider proof by contradiction for some of your steps)

- (i) $3n + 2$ is even
- (ii) $n + 5$ is odd
- (iii) n^2 even.