TEST 1

(Math 258 , B)

1. Let $P(x)$ be the statement "$x+1=2$." If the universe of discourse consists of the integers, what is the truth value of the following? Explain. (15 pts)

   (a) $P(0)$ (b) $\exists x P(x)$ (c) $\forall x P(x)$

2. Determine the truth value of the following statements, if the universe of discourse consists of all real numbers. Explain. (15 pts)

   (a) $\forall x \exists y (x = y^2)$ (b) $\forall x \exists y (xy = x)$ (c) $\exists x \exists y [(x+3y=1) \wedge (2x+6y=3)]$

3. Determine whether the following arguments are valid, giving counterexamples where necessary: (10 pts)

   (a) If x is a real number such that $x > 3$, then $x^2 > 9$.
   Suppose that $x \leq 3$, then $x^2 \leq 9$.

   (b) If x^2 is irrational, then x is irrational.
   Suppose x is irrational, then x^2 is irrational.

4. Use a truth table to show that the following argument is valid: (15 pts)

$$\begin{array}{c} \neg r \\ p \rightarrow r \\ q \rightarrow r \\ \hline \neg(p \wedge q) \end{array}$$

5. Prove, or disprove, that the product of two irrational numbers is irrational. (10 pts)
(*Note*: If you are disproving, provide a counterexample)

6. Prove that if $n^3 + 5$ is odd, where n is an integer, then n is even. (10 pts)
(*Hint*: Use proof by contradiction)

7. Prove that if a, b and c are real numbers, then $\min(a, \min(b, c)) = \min(c, \min(a, b))$.
(*Hint*: Use proof by cases) (10 pts)

8. Show that, for integers, the following statements are equivalent: (15 pts)
(*Hint*: consider proof by contradiction for some of your steps)

(i) $3n + 2$ is even
(ii) $n + 5$ is odd
(iii) n^2 even.