TEST 2

(Math 140)

1. Find the angle between the vectors $\vec{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Are these two vectors parallel? Explain. (10 pts)

2. Show that the vectors $\vec{u} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ are linearly independent. (10 pts)

3. Show that $M_{2\times 2}(R)$ is spanned by the following vectors: (10 pts)

$$\vec{E}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \vec{E}_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \vec{E}_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \vec{E}_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

4. Given $A = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 3 & -1 \end{pmatrix}$, find $AB + B^2 - 6I$. (10 pts)

5. Given $A=\begin{pmatrix}1 & 2\\ 1 & 4\end{pmatrix}$, find A^{-1}. (10 pts)

6. Show that the following map is a <u>linear</u> map: (20 pts)

$$T: R^2 \to R^2$$
$$T\begin{pmatrix}u_1\\ u_2\end{pmatrix}=\begin{pmatrix}u_1\\ -u_2\end{pmatrix}$$

7. Identify the <u>type</u> of linear map the following represent: (20 pts)
(*Note*: By type, we mean whether is a reflection, projection or some other kind of linear map)

(a) $A=\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$ (b) $B=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$ (c) $C=\begin{pmatrix}\sqrt{3}/2 & -1/2\\ 1/2 & \sqrt{3}/2\end{pmatrix}$ (d) $D=\begin{pmatrix}1/2 & -1/2\\ -1/2 & 1/2\end{pmatrix}$

8. Find the matrix that changes the basis $\vec{v}_1=\begin{pmatrix}2\\ 1\end{pmatrix}$, $\vec{v}_2=\begin{pmatrix}1\\ 3\end{pmatrix}$ into $\vec{u}_1=\begin{pmatrix}1\\ 1\end{pmatrix}$, $\vec{u}_2=\begin{pmatrix}0\\ 1\end{pmatrix}$.
(*Note*: Do <u>not</u> solve any system to find the relevant coefficients. They are easy to find) (10 pts)