

TEST 3

(Math 140)

1. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} -1 & 1 \\ 1 & 2 \end{pmatrix}$. (20 pts)

2. Find the volume of the parallelepiped S determined by the vectors $\vec{u} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$
and $\vec{w} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$. Also, find the area of the parallelogram determined by \vec{u} and \vec{v} . (20 pts)

3. Find the parametric equations of the line l that passes through $P = (1,1,3)$ and is perpendicular to the plane M with equation $2x + 3y - z = 2$. (20 pts)

4. Identify the type of linear map the following represent: (20 pts)
 (Note: By type, we mean whether is a reflection, projection or some other kind of linear map)

$$(a) A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \quad (b) B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$(c) C = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (d) D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

5. Rotate the vector $\vec{u} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ around the vector $\vec{a} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$ by 180° . (20 pts)

(Note: Use the following matrix: $A = \begin{pmatrix} a_1^2 + c(1 - a_1^2) & a_1 a_2(1 - c) - a_3 s & a_1 a_3(1 - c) + a_2 s \\ a_1 a_2(1 - c) + a_3 s & a_2^2 + c(1 - a_2^2) & a_2 a_3(1 - c) - a_1 s \\ a_1 a_3(1 - c) - a_2 s & a_2 a_3(1 - c) + a_1 s & a_3^2 + c(1 - a_3^2) \end{pmatrix}$)