Test 4

(Math 258 , A)

1. Show that $k\binom{n}{k} = n\binom{n-1}{k-1}$, where $n, k \in \mathbf{N} - \{\mathbf{0}\}$ (10 pts)

2. What is the coefficient of $x^5 y^6$ in $(2x+y)^{11}$, without doing the expansion? (10 pts)

3. What is the probability that the sum of the numbers on two dice is even when they are rolled? (10 pts)

4. Find the probability of each outcome when a loaded die is rolled, if a 3 is twice as likely to appear as each of the other five numbers on the die. (10 pts)

5. Given a family with two children, what is the probability that both children are girls if the older child is a girl? (10 pts)

6. Determine whether the relation $R = \{(x, y) \in \Re \times \Re \mid x + y = 0\}$ is: (20 pts)

   (a) reflexive (b) symmetric (c) antisymmetric (d) transitive

7. Given the relations: $R_1 = \{(x, y) \in \Re \times \Re \mid x \leq y\}$, $R_2 = \{(x, y) \in \Re \times \Re \mid x < y\}$ and $R_3 = \{(x, y) \in \Re \times \Re \mid x \neq y\}$, find the following: (20 pts)
   (*Note*: Show all relevant work)

   (a) $R_1 \cup R_2$ (b) $R_1 \cap R_3$

   (c) $R_2 - R_3$ (d) $R_1 \circ R_3$

8. A relation R on as set A is called irreflexive if and only if $\forall a \in A, (a, a) \notin R$. Can a relation on a set be neither reflexive nor irreflexive? (10 pts)
   (*Note*: Whether your answer is **yes** or **no**, make sure you provide an example)