Test 4

(Math 258 , B)

1. Show that $\binom{n+1}{k} = (n+1)\dfrac{\binom{n}{k-1}}{k}$, where $n, k \in$ **N – {0}** (10 pts)

2. What is the coefficient of $x^7 y^6$ in $(x-2y)^{13}$, without doing the expansion? (10 pts)

3. If a fair coin is tossed 6 times, what is the probability of getting exactly 4 heads? (10 pts)

4. Find the probability of each outcome when a loaded die is rolled, if a 3 is twice as likely to appear as each of the other five numbers on the die. (10 pts)

5. Two dice are rolled. Find the probability that the sum is 10 or greater. (10 pts)

6. Determine whether the relation $R = \{(x, y) \in \Re \times \Re \mid x = 2y\}$ is: (20 pts)

   (a) reflexive (b) symmetric (c) antisymmetric (d) transitive

7. Given the relations: $R_1 = \{(x, y) \in \Re \times \Re \mid x \leq y\}$, $R_2 = \{(x, y) \in \Re \times \Re \mid x < y\}$ and $R_3 = \{(x, y) \in \Re \times \Re \mid x \neq y\}$, find the following: (20 pts)
   (*Note*: Show all relevant work)

   (a) $R_1 \cup R_2$ (b) $R_1 \cap R_3$

   (c) $R_2 - R_3$ (d) $R_1 \circ R_3$

8. A relation R on as set A is called irreflexive if and only if $\forall a \in A, (a, a) \notin R$. Can a relation on a set be neither reflexive nor irreflexive? (10 pts)
   (*Note*: Whether your answer is **yes** or **no**, make sure you provide an example)