HOMEWORK 1

(Math 140)

1. A parallelogram with sides of equal length is called a **rhombus**. Show that the diagonals of a rhombus are perpendicular. (10 pts)

   (*Hint*: Use two vectors $\vec{u}$ and $\vec{v}$ as adjacent sides and build the rhombus. Then, find the diagonals in terms of those vectors)

2. The set $\mathbb{R}^+ = \{x \in \mathbb{R} \mid x \geq 0\}$, equipped with the following *addition* and *scalar multiplication*: (10 pts)

$$x + y = xy$$
$$\lambda \cdot x = x^{\lambda}$$

   becomes a vector space. In this vector space, show that:

   (a) $1 + 2 = 2$ (b) $0 \cdot 2 = 1$

3. What is the dimension of $\mathbb{C}$, the vector space of the complex numbers? Why? (10 pts)

   (*Note*: The scalars are taken to be in $\mathbb{R}$, i.e. they are real numbers)

4. Show that the polynomials $p_1(x) = 1 + x$, $p_2(x) = 3$, $p_3(x) = x + x^2$ form a basis for the vector space $P_2[x]$, the space of second-degree polynomials. (10 pts)

5. Given $A = \begin{pmatrix} 6 & 9 \\ -4 & -6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$ find: (30 pts)

   (a) A^2. What do you observe?
   (b) AB and BA. Are they equal?
   (c) $B^2 - A - 2I$.

6. Let A be a square matrix. Then, show that: (30 pts)

   (a) $A + A^t$ is symmetric
   (b) $A - A^t$ is antisymmetric
   (c) Any square matrix A can be written as the sum of a symmetric and an antisymmetric matrix.

   (*Hint*: For part (c), use parts (a) and (b) above)