Key

TEST 1

(Math 140)

1. Given the function $f(x)=\dfrac{2x}{x-2}$, find the following: (20 pts)

(a) Find the value of $f(4)$

+3 Substitution
+1 Reduction
+1 Answer

$f(4)=\dfrac{2(4)}{4-2}=\dfrac{8}{2}=4$

(b) Solve the equation $f(x)=1$

+2 Anything
+3 Setup
+1 Reduction
+1 Answer

$\dfrac{2x}{x-2}=1 \Rightarrow 2x=x-2$

$x=-2$

(c) Is the point $(\frac{1}{2},-\frac{2}{3})$ on the graph of f?

(*Note*: Do not do the graph)

+2 Setup
+1 Reduction
+1 Answer
+1 Yes

$f(\frac{1}{2})=\dfrac{2(\frac{1}{2})}{\frac{1}{2}-2}=\dfrac{1}{-\frac{3}{2}}=-\dfrac{2}{3} \Rightarrow$ Yes

(d) What is the domain of f?

$\{x\in\mathbb{R} \mid x\neq 2\}$

Either
+0
or
+5

2. Given the functions $f(x)=\sqrt{x+1}$ and $g(x)=\dfrac{x}{2}$, find the following: (10 pts)

I can't break this one up
+0 or +5
& Other situation will be based on % ...

(a) $(f\circ g)(x)=f(g(x))=f(\frac{x}{2})$

$=\sqrt{\frac{x}{2}+1}$

(b) $(g\circ f)(3)$

+2 g-sub
+2 f-sub
+1 Answer

$g(f(3))=g(\sqrt{3+1})$

$=g(2)=\dfrac{2}{2}=1$

3. Given the function $f(x)=2-5x$, find the following: (10 pts)

(a) Check whether f is one-to-one

+2 Setup
+2 Reduction
+1 Answer

$f(x_1)\stackrel{?}{=}f(x_2)$

$2-5x_1\stackrel{?}{=}2-5x_2$

$-5x_1\stackrel{?}{=}-5x_2$

$x_1=x_2$ ✓

(b) Find f^{-1}

+2 Setup
+2 Reduction
+1 Answer

$x=2-5y$

$x-2=-5y$

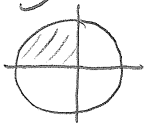
$\dfrac{x-2}{-5}=y$

$-\dfrac{x}{5}+\dfrac{2}{5}=y$

4. Do the following problems: (20 pts)

(a) If $\sin\theta=\dfrac{4}{5}, \dfrac{\pi}{2}<\theta<\pi$, find $\sec\theta$

+2 Identity
+3 $\cos^2\theta$
+2 $\cos\theta$
+2 $\sec\theta$
+1 Sign



$\sin^2\theta+\cos^2\theta=1$

$(\frac{4}{5})^2+\cos^2\theta=1$

$\cos^2\theta=1-\frac{16}{25}=\frac{9}{25}$

$\Rightarrow \cos\theta=-\sqrt{\frac{9}{25}}=-\frac{3}{5}$

$\Rightarrow \sec\theta=-5/3$

($\cos\theta$, $\sec\theta$ are neg. in QII)

(b) Find the exact value of $\sec(\tan^{-1}\sqrt{3})$

+5 $\tan^{-1}\sqrt{3}$
+5 $\sec\theta$

Let $\tan^{-1}\sqrt{3}=\theta$

Want to find $\sec\theta=?$

$\tan^{-1}\sqrt{3}=\theta \Rightarrow \tan\theta=\sqrt{3}$

But $\sec^2\theta=1+\tan^2\theta$

$\Rightarrow \sec^2\theta=1+(\sqrt{3})^2$

$\Rightarrow \sec\theta=\pm 2$

5. Establish the identity $1-\dfrac{\cos^2\theta}{1+\sin\theta}=\sin\theta$. (10 pts)

Graded on % correct

+2 if you tried

$$\left(\frac{1+\sin\theta}{1+\sin\theta}\right)1-\frac{\cos^2\theta}{1+\sin\theta}=\frac{1+\sin\theta-\cos^2\theta}{1+\sin\theta}$$

$$=\frac{(\sin^2\theta+\cos^2\theta)+\sin\theta-\cos^2\theta}{1+\sin\theta}=\frac{\sin^2\theta+\sin\theta}{1+\sin\theta}$$

$$=\frac{\sin\theta(1+\sin\theta)}{1+\sin\theta}=\sin\theta$$

+1 Split
+2 Identity
+1 Reduction
+1 Answer

6. Find the exact value of the following: (10 pts)
(*Note*: Use the *sum* and *half-angle* formulas)

(a) $\cos 105°$

$\cos(60°+45°)$

$=\cos 60°\cos 45°-\sin 60\sin 45°$

$=\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)-\frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$=\frac{1}{2\sqrt{2}}-\frac{\sqrt{3}}{2\sqrt{2}}=\frac{1-\sqrt{3}}{2\sqrt{2}}$

(b) $\tan\frac{\pi}{8}=\tan\left(\frac{\frac{\pi}{4}}{2}\right)=\pm\sqrt{\frac{1-\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}}$ +1 Split, +2 Identity, +1 Reduction, +1 Answer

$=\pm\sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}}=\pm\sqrt{\frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}}}}=\pm\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$

7. Establish the identity $\cot 2\theta=\dfrac{\cot^2\theta-1}{2\cot\theta}$. (10 pts)

Based on % correct

$$\frac{\cot^2\theta-1}{2\cot\theta}=\frac{\frac{\cos^2\theta}{\sin^2\theta}-\frac{1}{1}}{\frac{2\cos\theta}{\sin\theta}}=\left(\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}\right)\left(\frac{\sin\theta}{2\cos\theta}\right)=\frac{\cos^2\theta-\sin^2\theta}{2\sin\theta\cos\theta}$$

$$=\frac{\cos 2\theta}{\sin 2\theta}=\cot 2\theta$$

8. Solve the equation $\tan\theta=2\sin\theta$ in the interval $0\le\theta<2\pi$. (10 pts)

+3 solve for 0
+1 break
+2 sinθ
+4 cos$\theta=\frac{1}{2}$

$\tan\theta=2\sin\theta$

$\tan\theta-2\sin\theta=0$

$\frac{\sin\theta}{\cos\theta}-2\sin\theta=0$

$\sin\theta-2\sin\theta\cos\theta=0$

$\sin\theta(1-2\cos\theta)=0$

So, $\sin\theta=0$ or $\cos\theta=1/2$

$\Rightarrow \sin\theta=\sin 0$

$\Rightarrow \theta=k\pi+(-1)^k 0$

For $k=0$, $\theta=0$

For $k=1$, $\theta=\pi$

For $k=2$, $\theta>2\pi$

$\Rightarrow \cos\theta=\cos\pi/3$

$\Rightarrow \theta=2k\pi\pm\pi/3$

For $k=0$, $\theta=\pi/3$

For $k=1$, $\theta=\frac{5\pi}{3}$