TEST 2

(Math 140)

1. Show that the vectors $\vec{u} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ are linearly independent. (10 pts)

2. Show that $M_{2\times 2}(R)$ is spanned by the following vectors: (10 pts)

$$\vec{E}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \vec{E}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \vec{E}_3 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \vec{E}_4 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

3. Given $A = \begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}$, find $AB + B^2 - 2I$. (10 pts)

4. Given $A = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$, find A^{-1}. (10 pts)

5. (a) Show that the following map is a linear map: (30 pts)

$$T : R^2 \to R^2$$
$$T\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_1 + u_2 \\ -u_2 \end{pmatrix}$$

(b) Find Ker(T) .

6. Identify the type of linear map the following represent: (20 pts)
(*Note*: By type, we mean whether is a reflection, projection or some other kind of linear map)

(a) $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ (b) $B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ (c) $C = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$ (d) $D = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$

7. Find the matrix that changes the basis $\vec{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ into $\vec{u}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{u}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(*Note*: Do not solve any system to find the relevant coefficients. They are easy to find) (10 pts)