TEST 2

(Math 250 B)

1. (a) Show that the following map is a <u>linear</u> map: (20 pts)

$$T : R^2 \to R^2$$
$$T(x, y) = (x + y, x)$$

(b) Is the following map linear?

$$T : R^2 \to R$$
$$T(x, y) = xy$$

2. (a) Explain why there is a unique linear map $T : R^2 \to R^2$ for which T(3,1) = (2,-4) and T(1,1) = (0,2). (30 pts)

(b) Find a formula for T above.

(c) Is there a linear map $T : R^2 \to R^2$ for which T(2,2) = (8,-6) and T(5,5) = (3,-2) ?
(*Hint*: Notice that (5,5) = 5/2(2,2) and explain why this causes a problem)

3. Consider the linear map defined by: (20 pts)

$$T : M_{2\times 2}(R) \to M_{2\times 2}(R)$$ É
$$T(A) = A - A^t$$

Show that: (a) Ker(T) = Symm(R)

(b) Im(T) = AntiSymm(R)

4. Let L be an invertible $n \times n$ matrix and consider the linear map defined by:

$$T : M_{n\times n}(R) \to M_{n\times n}(R)$$ É
$$T(A) = LA$$

Show that T is one-to-one and onto. (20 pts)

5. Consider the linear map (“differentiation”) defined by: (10 pts)

$$D : P_n[x] \to P_{n-1}[x]$$ É
$$D(p(x)) = p'(x)$$

Find Ker(D) and use that to conclude that D is onto.

(*Hint*: Use the *Dimension Formula* for the 2nd half of the problem)