

Test 4

(Math 140)

1. Given $A = \begin{pmatrix} -1 & 2 & 0 \\ 1 & -3 & 2 \\ 3 & -9 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 & 0 \\ 0 & -2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$, examine which of the two is invertible without calculating any inverses. (10 pts)

2. True or False: $|A + B| = |A| + |B|$. If your answer is False, make sure you provide a counterexample. (10 pts)
(Note: You can use 2x2 matrices for a counterexample)

3. Consider the triangle with vertices $A = (1,2,0)$, $B = (0,0,0)$, and $C = (2,0,0)$.
- (a) Find the 4×4 matrix that first rotates the triangle ABC by 180° about the y -axis, and then translates it by $(0,1,0)$.
- (b) Use the above matrix to find the coordinates of the shifted triangle.

- (c) Make all relevant graphs that show the original triangle, the rotation, and the shifted triangle. (30 pts)

4. Given the basis $\vec{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$, $\vec{w} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, examine whether it is an orthonormal

basis. If your answer is **no**, then use the *Gram-Schmidt Orthogonalization Process* to construct an orthonormal basis from the basis above. (30 pts)

(Note: Take the inner product to be the usual **dot** product)

5. (a) True or False:

- (i) A Bezier curve is linear if and only if all control points lie on the curve.
- (ii) Bezier curves are not preserved under affine transformations.

- (b) Find the point on the Bezier curve determined by $A = (-1,2)$, $B = (1,2)$, and $C = (4,0)$ that corresponds to $t = 1/4$. (20 pts)