TEST 1

(Math 250 B)

1. (a) Show that the midpoint of $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is given by: (20 pts)

$$P_M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

(*Hint*: Consider any of the triangles generated by the position vectors $\overrightarrow{OP_1}, \overrightarrow{OP_2}$ and $\overrightarrow{OP_M}$)

(b) Use the above formula to find the midpoint of $P_1 = (1,-1)$ and $P_2 = (\frac{1}{2}, 3)$.

2. (a) Find the equation of the line L through the points $P_1 = (-1,1,2)$ and $P_2 = (0,2,1)$.
Does the point $Q = (-1,1,-1)$ lie on L ? (20 pts)

(b) Find the equation of the plane M that contains the point $P = (1,-1,2)$ and the line $L: \ x = t, y = 1 + t, z = -3 + 2t$.

3. The set $\mathbb{R}^+ = \{x \in \mathbb{R} \mid x \geq 0\}$, equipped with the following *addition* and *scalar multiplication*: (20 pts)

$$x + y = xy$$
$$\lambda \cdot x = x^\lambda$$

becomes a vector space. In this vector space, show that:

(a) $1+2=2$ (b) $0 \cdot 2 = 1$

4. Let $W = \{p(x) \in P_2[x] \mid p(5) = 0\}$. Show that W is a subspace of $P_2[x]$. (10 pts)

(*Note*: W is the set of all quadratic polynomials that have 5 as a root)

5. Let $W = \{A \in M_{2\times2}(\mathbb{R}) \mid \det(A) = 0\}$. Is W a subspace of $M_{2\times2}(\mathbb{R})$? (10 pts)

If your answer is *no*, provide a counterexample.

(*Note*: W is the set of all non-invertible matrices)

6. Show that the vectors $x^2, x+1, 1-x-x^3, 2$ span the vector space $P_3[x]$. (10 pts)

7. Show that the vectors $A_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, A_2 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ are linearly independent in $M_{2\times2}(\mathbb{R})$.

(10 pts)